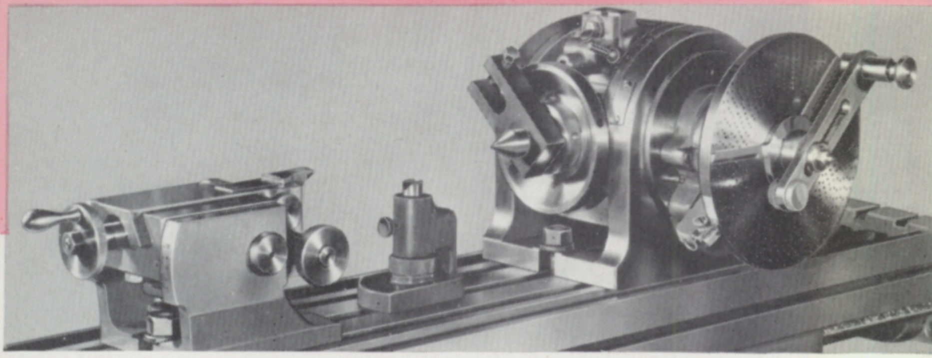
Cutting Gear Teeth

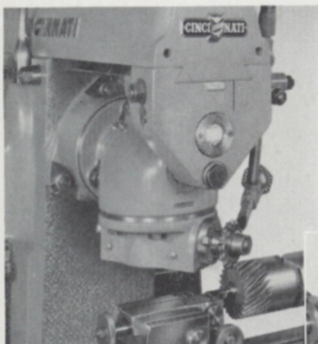
ON A MILLING MACHINE

THE CINCINNATI MILLING MACHINE CO.

Equipment for CUTTING GEAR TEETH
ON CINCINNATI MILLING MACHINES

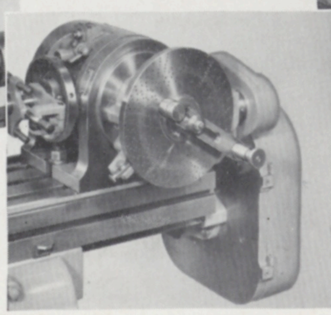


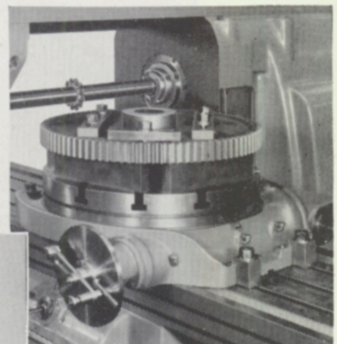
Universal Dividing Head . . . for accurate spacing of gear teeth, available in 10″, 12″ and 14″ sizes. May be equipped with Wide Range Divider at the factory.



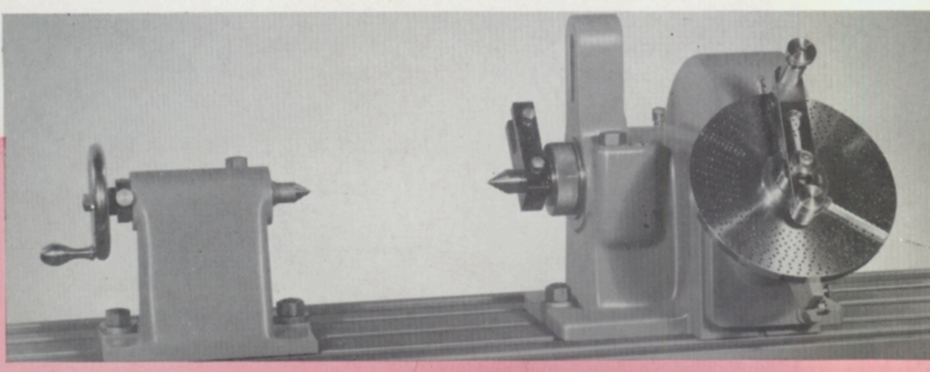
Above: Universal Spiral Milling Attachment . . . for cutting helical gears on plain or universal machines.

Below: Enclosed Driving Mechanism ... for cutting helical gears and worms with the Universal Dividing Head.





Above: Circular Milling Attachment equipped with Index Plate . . . for cutting large diameter spur gears.



Gear Cutting Attachment . . . for cutting spur gears. Also available as a Spiral Milling Head for cutting helical gears and worms. Both built in 12″ and 16″ sizes.

CUTTING GEAR TEETH ON A MILLING MACHINE

THIS booklet contains instructions on cutting worms and worm-wheels, and spur, helical and bevel gear teeth on a milling machine. Worms, spur gears and helical gears may be milled at a moderate rate of production. The milling machine is only recommended for bevel gears and worm-wheels when the usual specialized machines for these operations are not available or where only a single gear is required for experimental machinery.

Index tables and tables of leads have not been included, because they depend to some extent upon the age and style of the equipment. If not available, these tables may be obtained, in chart or booklet form, by writing to the address below, giving your CINCINNATI machine serial number and details of your gear cutting equipment.

The Cincinnati Milling Machine Co.
Cincinnati 9, Ohio, U. S. A.

CINCINNATI

PATENT NOTICE

The machines and attachments illustrated and described in this booklet are protected by issued and pending United States and Foreign Patents. The design and specifications of the machines illustrated herein are subject to change without notice.

CONTENTS

MILLING RACK TEETH

Rack teeth, either straight or inclined with respect to the rack blank, can be milled by using the rack cutting and rack indexing attachments. In the latter case, the rack is known as a spiral rack and can be cut only on universal milling machines (Figure 1). The following example illustrates the milling of this type of rack.

EXAMPLE 1: Milling the teeth in a spiral rack shown in **Figure 1**, at an angle of 23° 37′. The work material is annealed **S.A.E. 1112** steel (150—160 **B.H.N.**) and is milled according to the following specifications.

Diametral pitch = 8.

Addendum $= \frac{1}{8} = 0.125$ in.

Dedendum $= \frac{1}{8} = 0.125$ in.

Clearance $= \frac{0.157}{8} = 0.0197$ in.

Whole depth = 0.2696 in.

Normal circular pitch = 0.3927 in.

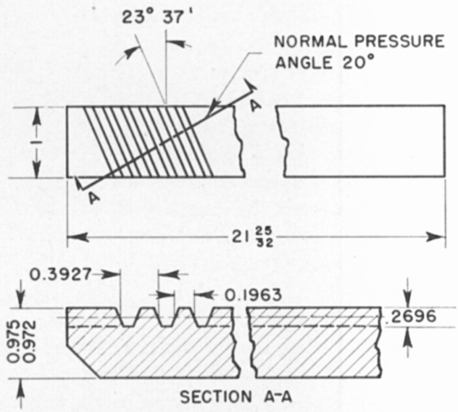


Figure 1—Dimensions of a spiral rack.

Normal tooth thickness at pitch line $= \frac{1.5708}{8} = 0.19635$ in.

Amount to index per tooth $= \frac{0.3927}{\cos 23° \ 37'} = 0.42859$ in.

Normal pressure angle = 20°.

Selection of Set-Up. The rack blank is held in a *rack vise* clamped to the table of a universal milling machine (Figure 2). The table of the machine is then swiveled in a clockwise direction to the required angle of 23° 37′. This will place the rack blank so that the rack teeth will be parallel to the saddle cross feed, and also to the milling cutter mounted on the *rack cutting attachment*.

In this position, the machine table moves parallel to the linear pitch of the rack teeth. Therefore, to space the teeth at the normal pitch of

0.3927 in., it is necessary to index each tooth into position by moving the table a distance of $0.3927/\cos 23^\circ 37' = 0.42859$ in., equal to the linear pitch. The indexing operation is performed by means of the *rack indexing attachment* shown in Figures 2 and 3.

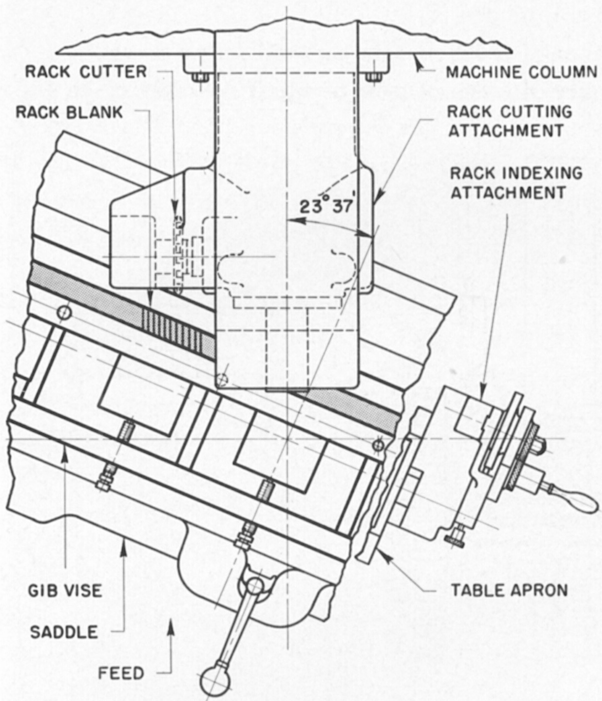


Figure 2 — Plan view of set-up for milling rack teeth at an angle of 23° 37′ by using the cross feed of the machine. The milling cutter is mounted on a rack-cutting attachment which places the cutter parallel to the motion of the saddle.

This attachment consists of a pair of change gears *A* and *B*, index plate *E* with two diametrically opposed notches, and gears *C* and *D* which rotate the table screw (Figure 3). Fourteen change gears (Table I, Page 56) provide a range of diametral pitches from 4 to 32, and table displacements from 0.250 in. to 0.03125 in.

Selection of Change Gears. The gear combination to use in any rack milling job is determined from the ratio between the lead *L* of the table screw and the pitch *P* of the teeth, measured in the direction of table motion, as expressed in the following formula:

$$\frac{L}{P} = \frac{A}{B} \qquad [1]$$

where:

L = lead of table screw, inches.

P = pitch in the direction of table motion or amount indexed per division, inches.

A = number of teeth of gear on lead screw drive.

B = number of teeth of gear on shaft of index plate E.

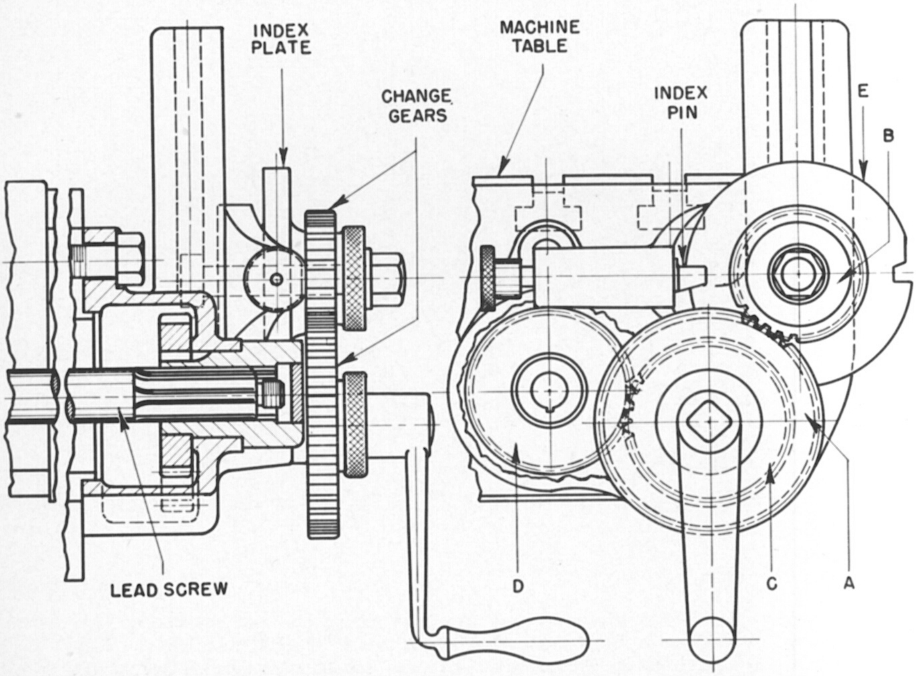


Figure 3—Rack indexing attachment mounted on the right end of a milling machine table.

In the present case, the table screw has a lead of 0.250 in., and the table displacement for each division is equal to the linear pitch, or 0.42859 in. Therefore:

$$\frac{A}{B} = \frac{L}{P} = \frac{0.25000}{0.42859} = 0.5833$$

To obtain the number of teeth in gears A and B, it is now necessary to reduce the ratio 0.5833 to a *common fraction*. This can be done by means of the *slide rule*, or by determining the quotients of *continued divisions*.

The object of this procedure is to eventually find a fraction having a ratio as close as possible to that of the original ratio, but in which the numerator and denominator are small numbers that *may be used directly* if they are equal to the number of teeth in the available change gears, or *can be transformed* into these numbers by means of suitable multipliers.

a. Slide Rule Method. With this method, the slide rule is set for the given ratio. Then the nearest matching whole numbers are read on the fixed and sliding, or *D* and *C* scales of the slide rule. These numbers are the numerator and denominator, respectively, of the common fractions approximating the value of the ratio.

A number of different fractions can usually be obtained with this method. Since the numerator and denominator of these fractions represent *the number of teeth in the change gears,* the only sets of fractions which can be used are those in which the numerator and denominator are equal to the number of teeth of any two gears in the set of gears which are a part of the indexing attachment (Table I, Page 56). The combination which has a ratio closest to the given ratio is then selected for the job.

If this procedure is carried out for the ratio 0.5833 of the example, the fraction 49/84 is obtained, and the change gears are $A = 49$ teeth, $B = 84$ teeth. The ratio of this fraction is equal to the above ratio to within the fourth decimal figure.

b. Continued Divisions Method. The number of teeth in gears *A* and *B* for the rack indexing attachment can also be determined by transforming the ratio:

$$\frac{L}{P} = \frac{0.25000}{0.42859} = \frac{25000}{42859} = 0.5833$$

into a series of common fractions by means of continued divisions, as follows:

In the first division, the denominator 42859 is divided by the numerator 25000 for only the whole part of the quotient. In the second division, the dividend is 25000, and the divisor is the remainder of the first division. In the third division, the dividend is the remainder of the first division,

and the divisor is the remainder of the second division. Hence, divisions are carried out by dividing each last remainder into the last divisor, and always for the whole part of the quotient, until the remainder of the last division is zero.

For convenience, the *continued divisions* are arranged as follows:

$$
\begin{array}{lllllllll}
25000\ ) & 42859\ (\ 1 & & & & & & & \\
 & \underline{25000} & & & & & & & \\
 & 17859\ ) & 25000\ (\ 1 & & & & & & \\
 & & \underline{17859} & & & & & & \\
 & & 7141\ ) & 17859\ (\ 2 & & & & & \\
 & & & \underline{14282} & & & & & \\
 & & & 3577\ ) & 7141\ (\ 1 & & & & \\
 & & & & \underline{3577} & & & & \\
 & & & & 3564\ ) & 3577\ (\ 1 & & & \\
 & & & & & \underline{3564} & & & \\
 & & & & & 13\ ) & 3564\ )\ 274 & & \\
 & & & & & & \underline{3562} & & \\
 & & & & & & 2\ ) & 13\ (\ 6 & \\
 & & & & & & & \underline{12} & \\
 & & & & & & & 1\ ) & 2\ (\ 2 \\
 & & & & & & & & \underline{2} \\
 & & & & & & & & 0
\end{array}
$$

From the quotients of these continued divisions, it now is possible to obtain a series of common fractions whose ratios gradually approach the value of the original fraction. This is done by means of a diagram like that in Table **A** (**Page 9**).

The quotients obtained in the continued divisions are listed in sequence in both the top and bottom lines of the diagram, under the column headings *A*, *B*, *C*, etc. To determine the numerators and denominators of the common fractions, the figures 1 and 0 are placed in the two spaces at the left end of the second and third lines, as shown, when the numerator of the original fraction is *smaller* than the denominator.

Table A

Tabulation for Determining Common Fractions when the Numerator of the Original Fraction is Smaller than the Denominator

Sequence:			*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	
Quotients			1	1	2	1	1	274	6	2	
	1	0	**1**	**1**	**3**	**4**	**7**	**1922**	**11539**	**25000**	Numerators
	0	1	**1**	**2**	**5**	**7**	**12**	**3295**	**19782**	**42859**	Denominators
Quotients			1	1	2	1	1	274	6	2	

The various common fractions inside the heavy border are then obtained, starting with the first quotient in column *A* and proceeding column by column to the right as follows:

To determine the *numerators* of these fractions, each quotient in succession in the *top line of Table A* is multiplied by the figure in the *space one column to the left in the numerator line, and the product is then added to the figure in the space two columns to the left in the numerator line.* The resulting figure, shown in heavy type, is now placed in the space immediately below the quotient from which it was obtained.

To illustrate, the first quotient obtained in the continued divisions is 1. The *first numerator* is obtained by multiplying the quotient 1 in column *A* of the top line by the number 0, found in the line below, one column to the left, and then adding the number 1, found two columns to the left in the numerator line, thus:

$$1 \times 0 + 1 = 1$$

This is the first numerator. It is placed in column *A* of the numerator line, under the quotient 1 from which it was derived.

The *second numerator,* in column *B*, and all of the others in succession are then similarly obtained. For example, the *third numerator,* in column *C*, is equal to the product of the quotient 2 and the number 1, below and to the left, plus the preceding number 1, two spaces to the left in the same line. The result is:

$$2 \times 1 + 1 = 3$$

The number 3 is now placed in the numerator line under the quotient 2 in column *C*.

If this operation is carried out for all quotients obtained in the continued divisions, the final result will be the numerator of the original fraction, as illustrated in Table **A**.

A similar procedure is used to determine the *denominators* of the common fractions, working *up* from the bottom line instead of *down* from the top line. These figures will then be listed in the denominator line of the table, immediately *above* the quotients from which they were derived.

Beginning at the left, the common fractions then are:

$$\frac{1}{1}, \frac{1}{2}, \frac{3}{5}, \frac{4}{7}, \frac{7}{12}, \text{etc.}$$

When the numerator of the original fraction is *greater* than the denominator, the numbers 1 and 0 at the left of the numerator and denominator lines are placed as indicated in Table **B**, reversing the position shown in Table **A** (Page 9).

If this change is applied in the present example, the number 42859 and all the rest of the numbers in the third line of Table **A** will become the numerators, and will be found in the numerator line of Table **B**. The number 25000 and all the rest of the numbers in the same line will become the denominators, and will now be found in the denominator line. This is illustrated in Table **B**:

Table B

Tabulation for Determining Common Fractions when the Numerator of the Original Fraction is Greater than the Denominator

Sequence:		*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	
Quotients		1	1	2	1	1	274	6	2	
0	1	**1**	**2**	**5**	**7**	**12**	**3295**	**19782**	**42859**	Numerators
1	0	**1**	**1**	**3**	**4**	**7**	**1922**	**11539**	**25000**	Denominators
Quotients		1	1	2	1	1	274	6	2	

As stated previously, the object of this procedure is to eventually find a fraction having a ratio *as close as possible to that of the original ratio,* but in which the numerator and denominator are small numbers that are either equal to or which, by means of suitable multipliers,

can be changed into numbers equal to the number of teeth in the change gears available.

On this basis, the fractions in columns *A* through *D*, inclusive, of Table A are discarded because their ratios are either ***lower or higher than the desired ratio,*** thus:

$$A.\ \frac{1}{1} = 1$$

$$B.\ \frac{1}{2} = 0.5$$

$$C.\ \frac{3}{5} = 0.6$$

$$D.\ \frac{4}{7} = 0.5711$$

The fractions in the *F, G* and *H* columns are also discarded because the numerators and the denominators of these fractions are ***greater than the number of teeth in the available change gears.***

Thus, the only fraction which is practical for use is fraction *E,* or 7/12. This has a ratio of 0.5833, which is equal to the given ratio to within the fourth decimal figure. When the numerator and denominator of this fraction are multiplied by 7, the result is 49/84, or the same fraction as obtained by means of the slide rule method.

Selection of Cutter. The cutter for milling the rack teeth is a special 4 in. diameter, zero rake angle, 18-tooth, high speed steel form relieved gear cutter, made for a normal pressure angle of 20° and having a tooth thickness at the circular pitch line of 0.1963 in., corresponding to the normal tooth thickness (Figure 1, Page 4). This cutter is usually known as a ***No. 1 gear cutter,*** and is made for cutting 135 teeth to a rack.

The shape of a gear tooth changes with the number of teeth in the gear. For example, the shape of a tooth in a 179 tooth gear is not exactly the same as in a 180 tooth gear. The difference is very slight, but it increases as the number of teeth decrease. For practical purposes, these variations can be ignored within certain limits. Eight cutters are usually sufficient to cut all gears from 12 teeth to a rack.

Table **II**, (Page 56), lists the numbers of standard involute gear cutters and the corresponding ranges of gear teeth which they are made to cut, as given by cutter manufacturers:

In the present example, the cutter is mounted on a rack milling attachment which places the cutter parallel to the cross feed.

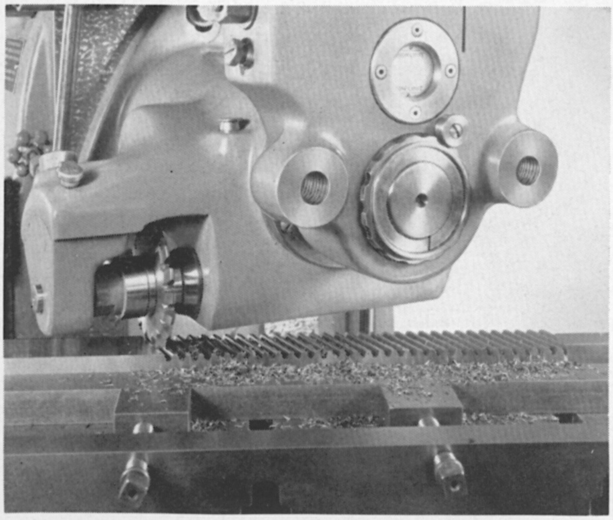


Figure 4 — Close-up view of the set-up used in milling the teeth of a spiral rack.

Locating the Workpiece with Respect to the Cutter. The workpiece is set vertically by hand feeding the milling machine knee up until the workpiece touches the rotating cutter. The dial on the knee adjustment is then set to zero reading and after clearing the cutter, the knee is raised 0.2696 in. to obtain the required depth of cut.

The workpiece is located longitudinally by moving the table and saddle until it lightly contacts one side of the cutter. The table lead screw dial is then set to zero reading and, after clearing the workpiece from the cutter, the table is moved to the right by an amount equal to approximately half the cutter thickness.

The set-up is now ready for milling the rack teeth by feeding the saddle in, and returning it to the starting position after each pass.

The next tooth is indexed into position by turning the crank of the rack indexing attachment until the index plate *E* has made one complete turn. Gear *B* on the index plate shaft will also make one complete turn, while gear *A* and the table lead screw will have completed 1.7143 revolutions. This corresponds to a table movement of 0.42857 in., which is very close to the value to be indexed. The same procedure is then repeated until the required number of rack teeth has been milled.

Before proceeding with the operation, however, the dimensions and spacing of the first few teeth should be checked. A close-up view of the set-up is shown in Figure 4.

MILLING SPUR GEAR TEETH

Gears which have straight teeth, cut parallel with the axis of rotation of the gear body, are known as *spur gears.* They are usually milled by using standard involute gear cutters of the arbor mounted type.

EXAMPLE 2: Milling the teeth of a 61-tooth, 10 pitch *spur gear* to full depth, or 0.225 in. (Figure 5). The work material is S.A.E. 3145 steel (180 B.H.N.).

Selection of Set-Up. The gear blank is placed on a mandrel held between centers on a universal dividing head. The dividing head is located on the table of a universal knee-and-column type milling machine.

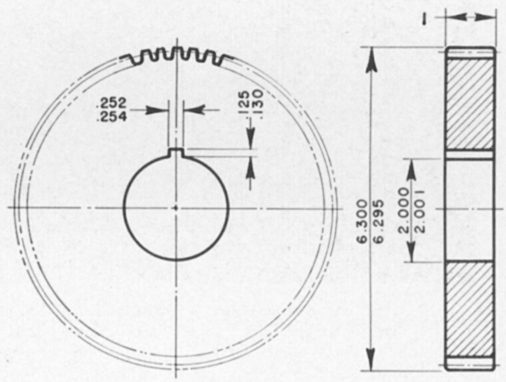


Figure 5—Dimensions of a 61-tooth 10 pitch spur gear.

Selection of Cutter. A high speed steel *No. 2 involute gear cutter* (Table II, Page 56) having a 10 diametral pitch and a pressure angle of 20° is mounted on the milling machine arbor (Figure 6). The cutter has a 3 in. outer diameter, a $1\frac{1}{4}$ in. hole diameter, and will cut gears having from 55 to 134 teeth. The cutting speed and feed per tooth are selected according to the type and material of the cutter and the material of the workpiece.

The circle of holes to use in indexing each tooth is obtained from the following formula:

$$t = \frac{N}{D} \qquad [2]$$

where:

t = number of complete turns and/or fraction of a turn of the index crank.

N = number of turns of index crank for one revolution of dividing head spindle or workpiece. This is equal to 40 turns in the dividing head.

D = number of divisions required in the workpiece.

In this case, $N = 40$ and $D = 61$. Hence from Formula 2 (Page 13):

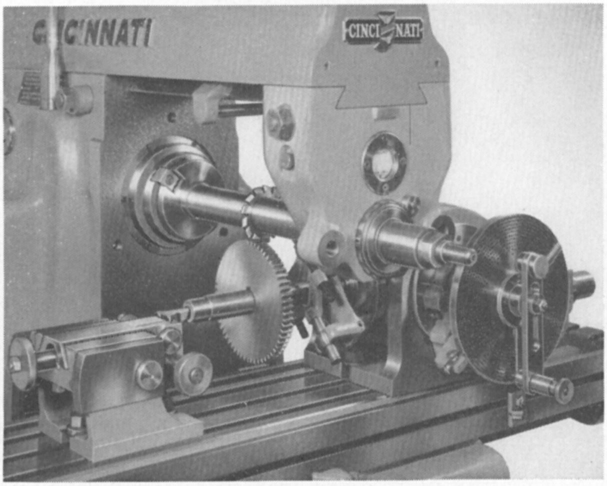


Figure 6—Set-up showing the use of a high number index plate in milling the spur gear illustrated in Figure 5.

$$t = \frac{40}{61}$$

A 61-hole circle would be required for this operation. But this number of holes is not available in any circle on the standard index plate (Table **III**, Page 56). It is therefore necessary to find a *new fraction* in which the denominator is equal to the number of holes in one of the circles on the standard plate, or, if the approximation is not satisfactory, to a circle of holes in the high number index plates.

If the numerator and denominator of 40/61 are multiplied by 3, this fraction is changed to 120/183. Inspection of Table **IV** (Page 57) reveals that side **B** of high number index plate No. 1 includes a 183-hole circle, as required for this indexing operation.

Therefore, the No. 1 high number index plate is mounted on the universal dividing head in place of the standard plate. Each of the 61 teeth is successively positioned for milling by indexing 120 spaces on the 183-hole circle. After centering the gear blank on the cutter and locating it vertically with respect to the cutter periphery, the knee is set for a depth of cut of 0.225 in., equal to the full depth of the tooth.

MILLING BEVEL GEAR TEETH

Bevel gears are characterized by a variable pitch diameter, tooth depth, and thickness.

Special form relieved milling cutters are used for milling bevel gears. These cutters are made to cut gears having a face width of not greater than one-third of the distance from the back of the gear to the apex of the cone.

The contour of the gear cutter teeth is made for the large end of the bevel gear, with the tooth depth sufficiently extended radially to permit cutting the flank of the teeth to the size required at this end.

It therefore follows that the shape of the teeth at any other section of the gear is only an approximation of the correct form or shape for that particular section. By following the proper procedure for milling bevel gear teeth, however, it is possible to approximate the dimensions and form of the teeth within satisfactory limits of accuracy for all ordinary applications.

Since the tooth space at the pitch diameter is narrower at the small end than at the large end of the gear, a gear cutter should be selected having a tooth thickness at the pitch line which will not exceed the dimensions of the tooth space at the small end of the gear.

Therefore, the thickness of the gear cutter tooth at the pitch line will be considerably less than the tooth space dimensions at the end of the gear. Consequently, the required dimensions of the gear teeth are obtained by removing additional stock from the flank of the teeth after the preliminary gashing operation.

The procedure used in milling straight bevel gears is illustrated in the following example:

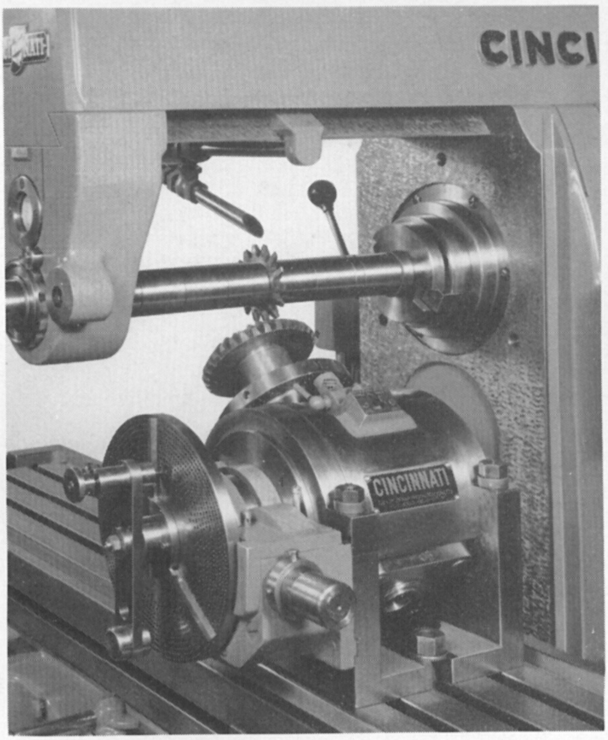


Figure 7—Set-up used in milling a bevel gear.

EXAMPLE 3: Milling the teeth of a 30-tooth, 6 diametral pitch, straight tooth bevel gear (45° Pitch Cone Angle, Figure 8) having a pressure angle of 14½°. Material: S.A.E. 3145 steel (175-180 B.H.N.).

The following information is obtained from bevel gear formulae, Page 49:

Pitch cone radius = 3.535 in.
Pitch diameter at large end = 5.000 in.
Pitch diameter at small end = 3.585 in.
Circular pitch at large end = 0.5236 in.
Circular pitch at small end = 0.3756 in.
Tooth thickness and tooth space at large end = 0.2618 in.
Tooth thickness and tooth space at small end = 0.1878 in.
Whole depth of tooth at large end = 0.3595 in.
Whole depth of tooth at small end = 0.2588 in.
Addendum at large end of gear = 0.166 in.
Addendum at small end of gear = 0.1195 in.
Dedendum + clearance at large end of gear = 0.193 in.
Dedendum + clearance at small end of gear = 0.139 in.

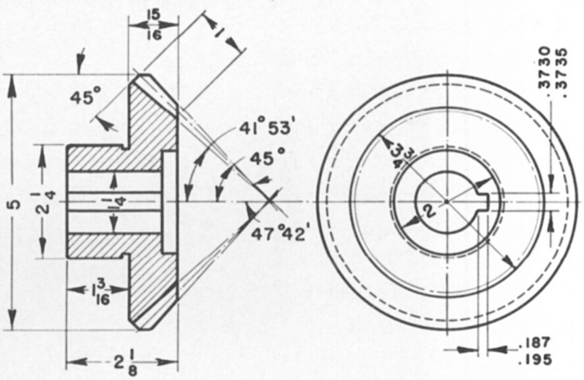


Figure 8—Dimensions of a straight tooth bevel gear.

Since all of the tooth parts at the small end of the gear are in exact proportion to those at the large end, any dimension at the small end can be obtained by multiplying the dimensions at the large end by the ratio C_s/C_r of the respective cone radii. In the present example, $C_r = 3.535$ in. and $C_s = 3.535 - 1 = 2.535$ in. Hence the ratio:

$$\frac{C_s}{C_r} = 0.72$$

The dimensions at the small end of the gear listed in the foregoing have been obtained by means of this procedure.

Selection of Set-Up. The gear blank is held on the spindle of a universal dividing head which is aligned in the direction of the table T-slots of a knee-and-column type milling machine. The large end of the gear is located toward the dividing head (Figure 7, Page 15).

The gear blank is tilted to the cutting angle of 41° 53′ by swiveling the dividing head spindle in the vertical plane. The angle is read on the graduations provided on the swivel block (Figure 9).

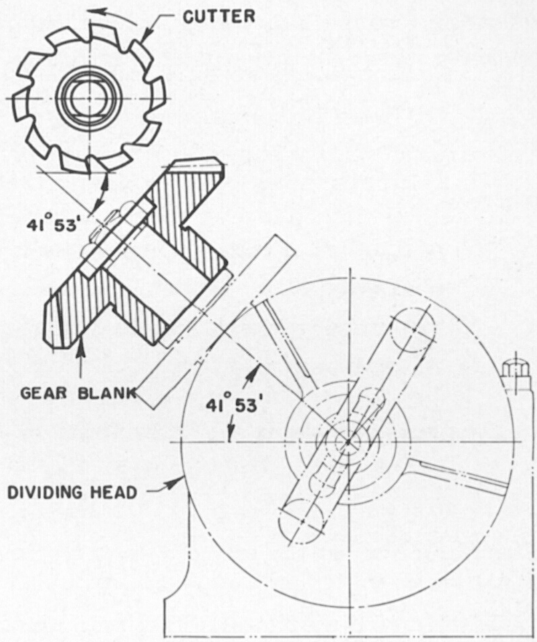


Figure 9—Elevation showing position of gear blank with respect to gear cutter when tilted at the cutting angle of 41° 53′.

Selection of Bevel Gear Cutter. Best results in cutting a bevel gear tooth are obtained if the gear cutter is selected not for the number of teeth that the bevel gear is to have, but rather for the teeth of an *imaginary spur gear* of an entirely different diameter than the bevel gear, and calculated by means of the following formula:

$$D_1 = \frac{P_d}{\cos\, b} \qquad [3]$$

where:

D_1 = pitch diameter of an imaginary spur gear, inches.

P_d = pitch diameter of bevel gear at the large end, inches.

b = pitch cone angle, degrees.

Since:

$$D_1 = \pi P_c N_c$$
$$P_d = \pi P_c N_g$$

where:

P_c = circular pitch, inches.

N_c = number of teeth in imaginary spur gear.

N_g = number of teeth in bevel gear.

Substituting the latter expressions for D_1 and P_d in Formula 3, the following formula is obtained, which permits calculating the number of teeth for which the gear cutter is selected:

$$N_c = \frac{N_g}{\cos\ b}$$

where:

N_c = number of teeth of imaginary spur gear for which gear cutter is selected.

N_g = number of teeth in actual gear.

b = pitch cone angle, degrees.

In the present example, $N_g = 30$ and $b = 45°$. Hence:

$$N_c = 43$$

Therefore, the cutter selected for this job is an arbor-mounted, high speed steel ***No. 3 gear cutter*** (Table **II**, Page 56) having a 3½ in. diameter and 6 diametral pitch, and made for milling gears with from 35 to 54 teeth.

The same cutter can be used for milling a mating gear of the *same size **and pitch,*** but a different cutter must be selected if the mating gear is of a ***different size.*** The teeth should preferably be milled by following the sequence of operations shown in Figure 10.

First Operation—Gashing the Teeth. In the first operation, the teeth are gashed after centering the bevel gear blank on the No. 3 gear cutter. This cutter is used in all subsequent operations.

By raising the knee, the blank is set for the depth of cut of 0.3595 in., equal to the full tooth depth at the large end of the gear. Each consecutive tooth is then positioned for milling by plain indexing.

The number of complete turns and fraction of a turn of the index crank, and the circle of holes to be used for indexing, are obtained by means of Formula 2 (Page 13) with $N = 40$ and $D = 30$, which is the number of teeth to be cut in the bevel gear. Hence:

$$t = \frac{40}{30} = 1\frac{1}{3} = 1\frac{18}{54}$$

Using the standard index plate (Table **III**, Page 56), the 30-hole circle would be satisfactory for this indexing operation. However, the 54-hole circle is selected because it will be useful when making the set-up for the subsequent operations.

Consequently, to index for each tooth, the index crank will be rotated one complete turn and 18 spaces on the 54-hole circle.

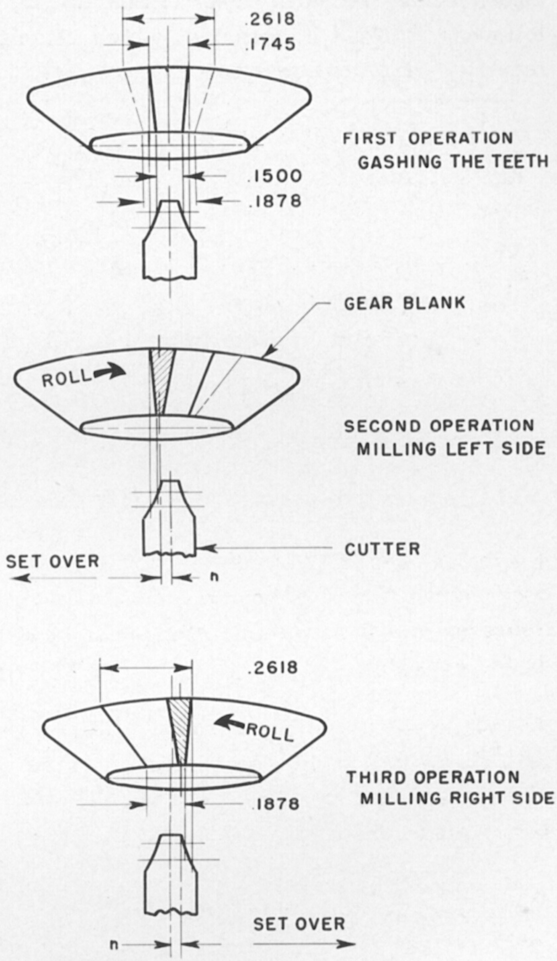


Figure 10—Sequence of operations when milling the teeth of a bevel gear.

Second and Third Operations — Milling Sides of Teeth. The width of the gashes produced by the cutter in the first operation, measured at the pitch line is 0.1745 in. and 0.150 in. at the large and the small ends of the gear, respectively. The correct dimensions for the finished gear are 0.2618 in. and 0.1878 in., respectively. In order to obtain these dimensions, an additional amount of stock must be removed on each side of the tooth space, as indicated by shading in **Figure 10**.

Determining the Angle of Roll. For this purpose, the blank must be rotated or rolled on its axis through an angle *C*, so that either line *AB* or line *CD* (view **X** in **Figure 11**—traces of the pitch line along the gear tooth face width) is placed in a direction parallel to the line *EF*. The latter connects the points corresponding to the dimensions of the gashes (**Figure 10**) at the pitch line, at the large and small ends of the gear.

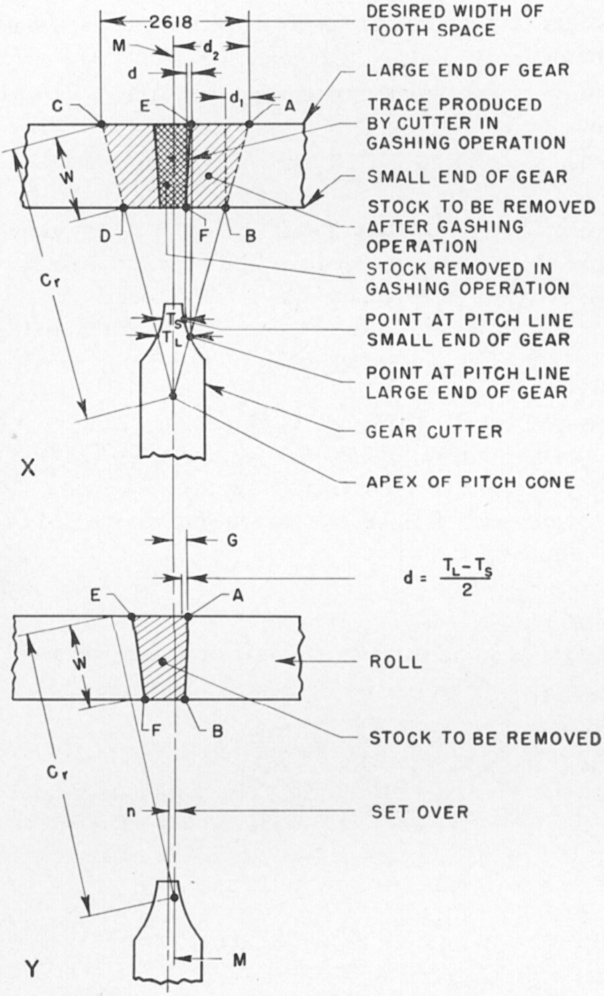


Figure 11—Geometrical relation between gear and cutter tooth to determine angle of roll.

Then point A, for example, will have moved from a distance d_2 to a distance G from the centerline M (view **Y** in Figure 11), and the distance d_1 between points A and B in view **X** of Figure 11 will change to the distance d between points E and F.

But d is half the difference between the chordal thicknesses T_L and T_s of the gear cutter (Figure 12), corresponding to the pitch line, at the large and small ends of the gear, respectively. From the geometry of Figure 11 (view **Y**), the distance G can be expressed as follows:

$$G = C_r \frac{d}{W}$$

But:

$$d = \frac{T_L - T_s}{2}$$

Hence:

$$G = C_r \frac{T_L - T_s}{2W} \qquad [5]$$

where:

G = distance between centerline of gear blank and point at pitch line of gear at large end, inches.

C_r = pitch cone radius, inches.

T_L = chordal thickness of gear cutter tooth at pitch line, at *large* end of gear, inches.

T_s = chordal thickness of gear cutter tooth at pitch line, at *small* end of gear, inches.

W = width of gear tooth face, inches.

The amount of gear blank roll, from the position in view **X** to that shown in view **Y** (Figure 11), is the difference between the circular distances d_2 and G of point A from centerline M of the blank. The distance d_2 is one-half of 0.2618 in., or one-quarter of the circular pitch P_c, and the distance G is obtained from Formula 5 (Page 20).

The corresponding angle of roll C in degrees is therefore obtained by dividing this difference by the pitch radius or one-half the pitch diameter of the large end of the gear, and then multiplying the result by the constant 57.3, which is the degrees of an arc corresponding to one radian. This is expressed in the following formula:

$$C = \frac{57.3}{P_d}\left(\frac{P_c}{2} - \frac{C_r}{W}(T_L - T_s)\right) \qquad [6]$$

where:

C = angle of roll, degrees.

P_d = pitch diameter at large end of gear, inches.

P_c = circular pitch at large end of gear, inches.

C_r = pitch cone radius at large end of gear, inches.

T_s, T_L = chordal thickness of gear cutter tooth corresponding to pitch line at small and large ends of gear, respectively, inches.

57.3 = degrees per radian.

W = width of gear tooth face, inches.

Calculating the Angle of Roll. All of the values to be used in Formula 6 are known, with the exception of the quantities T_s and T_L. These are obtained by direct measurement of the gear cutter tooth employed. Dimensions for the present example are given in Figure 12.

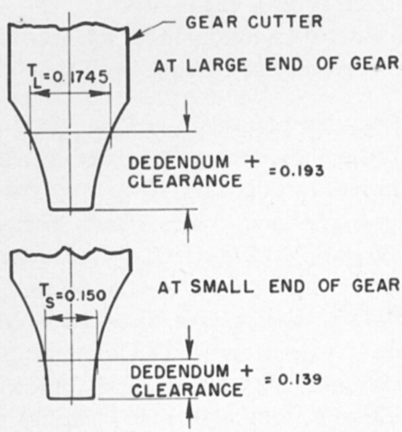


Figure 12 — Chordal thickness of the gear cutter, corresponding to the pitch line at the small and large ends of the gear, respectively.

Substituting the known values in Formula 6:

$$C = \frac{57.3}{5}\left(\frac{0.5236}{2} - \frac{3.535}{1}(0.1745 - 0.150)\right)$$

$$= \frac{57.3}{5}\left(0.2618 - 3.535 \times 0.0245\right)$$

$$= 11.46\left(0.2618 - 0.0866\right)$$

$$= 11.46 \times 0.175$$

$$= 2.007°$$

or very nearly 2°.

Indexing for the Angle of Roll. The angle of roll of 2° is obtained by plain indexing from the centered position of the blank. The number of spaces to index is obtained by dividing the angular distance between divisions by 9°, or the angular rotation of the spindle corresponding to one turn of the index crank, thus:

$$t = \frac{2}{9} = \frac{2 \times 6}{9 \times 6} = \frac{12}{54}$$

By indexing 12 spaces on the 54-hole circle, the blank will be rotated on its axis by the amount required to position one side of the teeth for producing the required tooth thickness.

After rolling the blank 2°, the subsequent teeth are, however, indexed from this new position by turning the index crank one full turn and 18 spaces on the 54-hole circle, as in the case of the gashing operation.

The *direction of roll* is not important, since the roll is reversed for milling the opposite sides of the teeth after all the teeth have been milled on one side. The only consideration is that *the direction of roll and the set-over must always be made in opposite directions* (Figure 10, Page 19).

Determining the Set-Over. After rotating the blank on its axis to the angle C, it must be set over by an amount n from the centered position. This is done to locate the blank so that the cutter will follow along the line AB, which is now parallel to the line EF produced by the cutter (view **Y**, Figure 11, Page 20). The set-over n is also calculated from the dimensions of the gear and cutter tooth, by means of the following formula:

$$n = \frac{T_L}{2} - \frac{T_L - T_s}{2} \frac{C_r}{W} \qquad [7]$$

Calculating the Set-Over. The set-over n is calculated by substituting the known values in Formula 7:

$$\begin{aligned} n &= \frac{0.1745}{2} - \frac{(0.1745 - 0.1500)\ 3.535}{2} \\ &= 0.0873 - \frac{0.0245 \times 3.535}{2} \\ &= 0.0873 - 0.01225 \times 3.535 \\ &= 0.0873 - 0.0433 \\ &= 0.044 \text{ in.} \end{aligned}$$

If the blank has been rolled 2° in a *counter-clockwise* direction (when looking at the spindle end of the dividing head), the machine table is moved out, or *away from the column,* 0.044 in.

Conversely, if the blank has been rolled 2° *clockwise* (when looking at the spindle end of the dividing head), the table is moved in *toward the* column of the machine 0.044 in., to offset the work by this amount with respect to the center position used in the gashing operation.

To set for milling the opposite side of the teeth after one side has been completed, the table is moved twice the amount of the set-over, or 0.088 in., and the blank is rolled twice the angle of roll, or 4°.

After milling two or three complete teeth, their thickness at the pitch line at the large and small ends of the gear should be measured. These measurements should be equal to the given tooth thickness at these ends. If not, the calculations and the set-up should be checked for possible errors.

Accuracy of Tooth Profile. The objective of this operation is to mill the gear teeth to the required thickness *at the pitch line* along the face width. The tooth form, however, will not be accurate throughout the length of the tooth face, especially at the small end of the gear.

Here the flank of the teeth on the addendum part of the profile may not curve sufficiently to avoid a slight interference with the mating teeth. This results from the fact that the gear cutter is made for a tooth form which is correct for the *large end of the teeth.* In any other section, it may vary as indicated by the broken lines shown in Figure 13, (Page 24).

Gears milled in accordance with the method described in the foregoing will be found to mesh satisfactorily. If necessary, however, a small amount of metal in the form of a triangular shape can be removed from the top of the teeth down to the pitch line at the small end, tapering off at the large end of the teeth, as indicated by the broken lines in Figure 13. This is done by rotating the blank through a small angle on the dividing head spindle, and then taking light cuts until satisfactory meshing conditions are obtained.

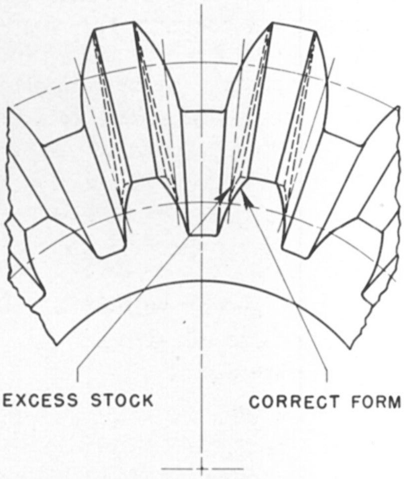


Figure 13—Effect of bevel gear cutter contour on the tooth shape.

This method of milling of bevel gears is especially convenient where the pitch cone radius is unusually large and regular gear cutting equipment does not have the range to accommodate the gear to be cut.

MILLING HELICAL GEAR TEETH

Helical gears are gears which have the teeth cut along a helical surface. They are usually milled by using standard involute cutters of the arbor-mounted type.

In order to mill helical gears, it is necessary to *rotate* and at the same time *feed* the workpiece while milling. One of the most generally used attachments for this type of work is the *universal dividing head,* driven from the table lead screw of the milling machine by means of change gears.

The change gears permit varying the ratio between the table feed rate and the revolutions per minute of the workpiece, and consequently the lead of the helical surface.

The dividing head indexing mechanism is used in spacing the helical teeth around the periphery of the workpiece as required.

The machine used for milling helical surfaces is usually a *universal knee-and-column type milling machine.* This permits swiveling the table, and consequently the workpiece located between centers of a dividing head and tailstock, to the required angle of swivel.

The same results may also be obtained if a plain knee-type machine is employed. In this case, however, additional equipment will be required. This consists of a universal spiral milling attachment which permits swiveling the cutter to the required angle of swivel.

When specifying the helix angle of helical gear teeth, the angle C (Figure 14) is preferred because this is also the angle used in setting up the blank for the milling operation. In other cases, as in the case of lead screws, the angle E is used to specify the helix angle of the threads. To avoid errors, it is therefore necessary to indicate clearly to which helix angle the given value should apply.

From the geometry of Figure 14, the following formulae are obtained:

$$E + C = 90 \qquad [8]$$

$$L = \pi D \tan E \qquad [9]$$

$$L = \pi D \cot C \qquad [10]$$

where:

E and C = helix angles, degrees.

L = lead of helix, inches.

D = diameter of the cylinder or blank, inches.

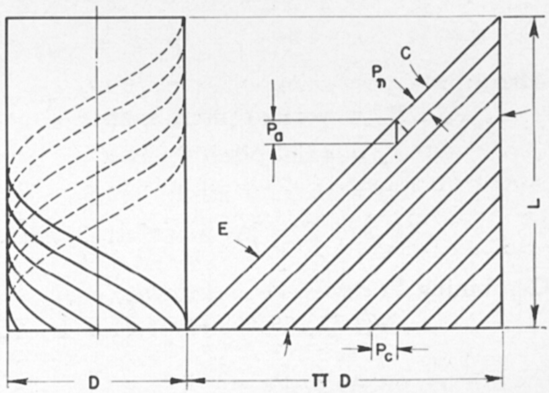


Figure 14—Axial, circular and normal pitch of equally spaced teeth.

If a helical gear is rolled on a plane surface, the traces of the teeth will line up in equally spaced parallel straight lines. *The axial distance P_a between consecutive teeth is the axial pitch, P_n is the normal pitch and P_c is the circular pitch of the teeth.* The circular pitch, normal pitch and axial pitch are measured as shown in Figure 14, at the pitch line of the gear.

The *normal pitch* measures the thickness of the gear cutter tooth, and the tooth thickness and tooth space of the gear at the pitch line.

The *circular pitch* is calculated by substituting the known pitch diameter of the gear for the diameter D, and the number of teeth in the gear for N; hence:

$$P_c = \frac{\pi P_d}{N} \qquad [11]$$

where:

P_c = circular pitch, inches.

P_d = pitch diameter of gear, inches.

N = number of teeth.

The normal and axial pitches P_n and P_a can be obtained from the circular pitch P_c and the helix angle, as follows:

$$P_n = P_c \cos C$$

or:

$$P_n = P_c \sin E \qquad [12]$$

and:

$$P_a = P_c \tan E$$

or:

$$P_a = P_c \cot C$$

where:

P_n = normal pitch, inches.

P_a = axial pitch, inches.

P_c = circular pitch, inches.

E and C = helix angles, degrees.

Combining Formula 11 and Formula 12:

$$P_n = \frac{\pi P_d}{N} \cos C \qquad [13]$$

but the normal diametral pitch is:

$$P_{nd} = \frac{N}{P_d \cos C} \qquad [14]$$

The ***diametral pitch*** P_{nd} of helical gears is specified by numerical values such as 5, 7 or 10 in the same way as the diametral pitch for spur gears. By combining Formulae 13 and 14, the following normal pitch formula results:

$$P_n = \frac{\pi}{P_{nd}} \qquad [15]$$

This is similar to the formula for spur gears. In the latter, the circular pitch is the normal pitch of the helical gear teeth.

Helical Gears with Shafts at Right Angles

Two helical gears with shafts *at right angles to each other* have different helix angles. Each angle is the complement of the other (Figure 15); hence:

$$C_1 + C_2 = 90 \qquad [16]$$

The center distance of the gears is the semi-sum of their respective pitch diameters, as expressed by solving Formula 14 for P_d and applying it to each gear. The result is the following center distance formula:

$$S = \frac{1}{2\,P_{nd}} \left(\frac{N_1}{\cos C_1} + \frac{N_2}{\cos C_2} \right) \qquad [17]$$

but from Formula 16:

$$C_2 = 90 - C_1$$

therefore:

$$S = \frac{1}{2\,P_{nd}} \left(\frac{N_1}{\cos C_1} + \frac{N_2}{\sin C_1} \right) \qquad [18]$$

where:

S = center distance between the two gears, inches.

P_{nd} = normal diametral pitch of gears.

N_1, N_2 = number of teeth in the gears.

C_1, C_2 = helix angles of the two gears, degrees.

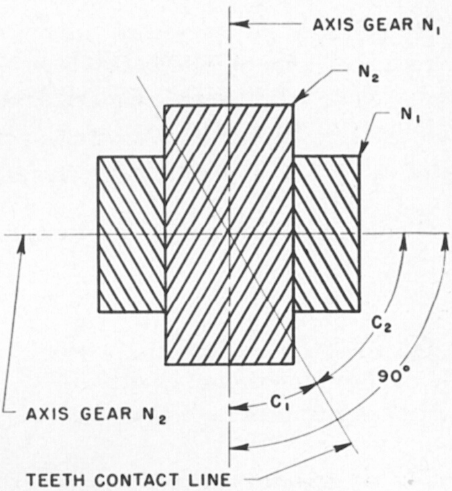


Figure 15—Helical gears with shafts at right angles.

Helical Gears with Parallel Shafts

When the shafts are *parallel* the angles C_1 and C_2 in Formula 16 become *equal,* because the helix angle is the same for both gears. The center distance Formula 18 changes to the following:

$$S = \frac{1}{2\,P_{nd} \cos C} (N_1 + N_2) \qquad [19]$$

Helical Gears with Shafts at an Angle of Less than 90°

Helical gears may have shafts *at an angle of less than 90°*. In such cases, the center distance between the meshing gears is determined by means of Formula 17, as in the case of gears with shafts at right angles.

Selection of the Cutter for Milling Helical Gears

Spur gear cutters for milling helical gears are selected for the ***hypothetical number of teeth in the section at right angles to the helix,*** rather than the actual number of teeth to be cut. This section is an ellipse (Figure 16). If the length of this ellipse is divided by the normal pitch in the gear, the result is the number of teeth for which the cutter should be selected.

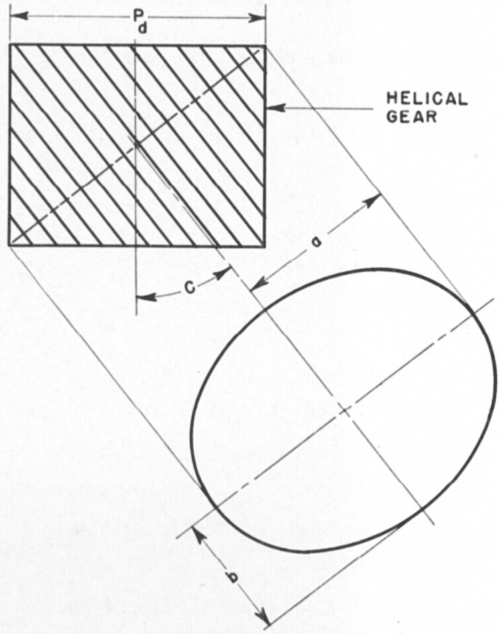


Figure 16—Elliptical section of a helical gear, used for determining the hypothetical number of teeth for which the gear cutter is selected.

The major and minor axes, a and b respectively, of the ellipse can be expressed in terms of the pitch diameter P_d and helix angle C, as follows (Figure 16):

$$a = \frac{P_d}{2 \cos C}$$

[20]

$$b = \frac{P_d}{2}$$

but the approximate length of the corresponding ellipse is:

$$M = \pi \ (a + b)$$

Substituting the expressions for a and b in Formula 20, the following result is obtained:

$$M = \frac{P_d \ (1 + \cos C)}{2 \cos C}$$

Now, by dividing M by the normal pitch P_n, the result is the hypothetical number of teeth N_c in the normal section:

$$N_c = \frac{M}{P_n} = \frac{\pi \ P_d \ (1 + \cos C)}{2 \ P_n \cos C}$$

Using the expression for P_n obtained from Formula 15 (Page 26) and the expression of P_d obtained by solving Formula 14 (Page 26), the preceding formula can now be written as follows:

$$N_c = \frac{N\,(1 + \cos C)}{2 \cos^2 C} \qquad [21]$$

This formula gives the hypothetical number of teeth which is used for selecting the gear cutter.

Another formula commonly used is the following:

$$N_c = \frac{N}{\cos^3 C} \qquad [22]$$

This formula is obtained by considering the radius of curvature R of the elliptical section normal to the helix (Figure 16), at a point corresponding to the minor axis b. Hence:

$$R = \frac{a^2}{b}$$

Substituting for a and b in this formula the expressions given in Formula 20:

$$R = \frac{P_d}{2 \cos^2 C}$$

also:

$$2\,\pi\,R = \frac{\pi\,P_d}{\cos^2 C}$$

Dividing this formula by Formula 14 (Page 26):

$$\frac{2\,\pi\,R}{P_n} = \frac{\pi\,P_d}{P_c \cos^3 C}$$

but:

$$\frac{2\,\pi\,R}{P_n} = N_c, \text{ and } \frac{\pi\,P_d}{P_c} = N$$

After substituting these values in the preceding formula, the latter becomes equal to Formula 22.

Formula 22 gives a slightly higher number of teeth than Formula 21. In the above formulae:

N_c = hypothetical number of teeth for which the gear cutter should be selected.

N = actual number of teeth in the gear.

C = helix angle, degrees.

Milling Helical Gears for Parallel Shafts

EXAMPLE 4: Milling a pair of helical gears for use on parallel shafts. The center distance is 8 in.; the gear ratio 2:1, the normal diametral pitch 5. The width of the gears is 1½ in. The teeth have the same helix angle but opposite "hands" of helix.

Selection of Set-Up. Helical gears are milled with the same equipment as used in milling helical and plain milling cutters. This equipment may consist of either a *universal knee-and-column type milling machine* with a dividing head and the dividing head driving mechanism, and an arbor to mount the gear cutter; or a *plain horizontal knee-and-column type milling machine* equipped with a universal spiral milling attachment.

When the latter equipment is used, the gear cutter, rather than the machine table, is set at the angle required to place it in the plane of the tangent to the helix of the teeth to be cut. In helical gears, the value of the helix angle is calculated from or related to the pitch diameter.

Number of Teeth and Helix Angle. If N is the number of teeth in the small gear, the number of teeth in the large gear will be 2 N. Substituting $N_1 = N$, $N_2 = 2\ N$, $P_{nd} = 5$, $C_1 = C_2 = C$, and $S = 8$ in. in the center distance Formula 18 (Page 27), the formula as it applies in the present example is as follows:

$$8 = \frac{2\ N + N}{2 \times 5 \times \cos C}$$

or:

$$80 = \frac{3\ N}{\cos C} \qquad [a]$$

In this formula, there are two unknowns: the number of teeth N in the small gear and the helix angle C of both gears. When one of these factors is assumed, the other can be calculated.

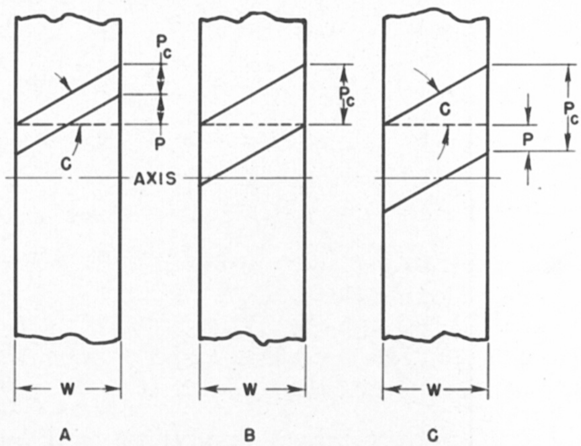


Figure 17—Overlap, no lap and underlap between teeth in helical gears.

The value of the helix angle C can be established on the basis of the

overlap, underlap or no lap between the teeth in the gear, as measured in a direction parallel to the gear axis (**Figure** 17).

The *overlap* is the distance P (as in view **A** of **Figure** 17), by which the axial projection of the *end* of one tooth overlaps the *beginning* of the next tooth; hence, from the geometry of view **A**, **Figure** 17:

$$P_c = W \tan C - P \qquad [23]$$

where:

P_c = circular pitch, inches.

P = overlap, inches.

W = width of the gear, inches.

C = helix angle, degrees.

Expressing the overlap P as a percentage K of the circular pitch:

$$P = K\ P_c$$

Formula 23 can be written thus:

$$P_c = \frac{W \tan C}{1 + K} \qquad [24]$$

When $K = 0$, the teeth have *no lap,* as in view **B** of **Figure** 17.

When the teeth have an *underlap P,* as in view **C** of **Figure** 17, **Formula** 23 changes as follows:

$$P_c = W \tan C + P \qquad [25]$$

Thus, in general, the conditions of overlap, no lap and underlap of the teeth in a helical gear can be determined by means of the following formula, which is obtained by combining **Formulae** 23, 24 and 25:

$$P_c = \frac{W \tan C}{1 \pm K} \qquad [26]$$

The plus sign in the denominator is used when the teeth overlap, while the minus sign is used with the teeth underlap. $K = 0$ when there is no lap.

Combining **Formulae** 11 (Page 25) and 14 (Page 26) and solving for $\mathbf{P_c}$:

$$P_c = \frac{\pi}{P_{nd} \cos C}$$

Hence, substituting this expression for P_c in **Formula** 24 and solving for the function of **C**:

$$\sin C = \frac{(1 \pm K)\, \pi}{P_{nd}\, W} \qquad [27]$$

In the present example, $W = 1\frac{1}{2}$ in. and $P_{nd} = 5$, and if the teeth have no lap, $K = 0$; hence:

$$\sin C = \frac{3.1416}{5 \times 1.5}$$

$$= 0.4188$$

and:

$$C = 24° \ 46'$$

Substituting this value of C in Formula a (Page 30), and solving for the number of teeth N of the small gear, the following result is obtained:

$$N = \frac{80 \times \cos 24° \ 46'}{3}$$

$$= \frac{80 \times 0.908}{3}$$

$$= 24.2$$

Assuming 24 teeth for the small gear, the large gear will then have $2\ N = 48$ teeth. Recomputing the helix angle, from center distance Formula 19 (Page 27), the correct value of the angle C is $25° \ 50'$. This helix angle is also the angle of swivel of the machine table.

Selecting the Gear Cutters. If the gear cutter for the *small gear* is selected by means of Formula 21 (Page 29), with $N = 24$ and $C = 25° \ 50'$:

$$N_c = \frac{24\ (1 + \cos 25° \ 50')}{2 \cos^2 25° \ 50'}$$

$$= \frac{24 \times 1.9}{2 \times 0.81}$$

$$= 28$$

A No. 4, 5 diametral pitch gear cutter (Table II, Page 56) made to cut 26 to 34 teeth is the cutter to use for cutting the teeth of the small gear.

The *large gear* has 48 teeth and the same helix angle as the small gear. Hence, using Formula 21 (Page 29):

$$N_c = \frac{48\ (1 + \cos 25° \ 50')}{2 \times \cos^2 25° \ 50'}$$

$$= \frac{48 \times 1.9}{2 \times 0.81}$$

$$= 56$$

Here it is found that a No. 2, 5 diametral pitch gear cutter which cuts a range of teeth from 55 to 134 should be used for milling the teeth of the large helical gear.

In this case, the same results would be obtained by determining the value of N_c by means of **Formula** 22 (Page 29).

Formulae 21 and 22 give results which are satisfactory under general conditions, but should be considered as an approximation subject to corrections, especially when milling gears to close tolerances.

Computing the Lead. The lead is calculated by means of **Formula** 10 (Page 25), using the pitch diameter of the gear and the value of the helix angle at this diameter. Combining **Formula** 10 and **Formula** 14 (Page 26):

$$L = \frac{\pi N}{P_{nd} \sin C} \qquad [28]$$

For the small gear, $N = 24$, $P_{nd} = 5$ and $C = 25° \; 50'$. Hence:

$$L = \frac{3.1416 \times 24}{5 \times \sin 25° \; 50'}$$

$$= \frac{3.1416 \times 24}{5 \times 0.435}$$

$$= 34.668 \text{ in.}$$

For the large gear, the lead $= 2 \times 34.668 = 69.336$ in.

Change Gears for the Dividing Head Drive. Since $L = 34.668$ in., the change gears for the standard driving mechanism are calculated as follows:

$$\frac{A \times C}{B \times D} = \frac{34.668}{10} = \frac{8667}{2500}$$

This is simplified to the fraction 52/15. This fraction has a ratio of 34.666, which gives a sufficiently close approximation to the calculated lead.

By factorizing this fraction:

$$\frac{52}{15} = \frac{13 \times 4}{5 \times 3} = \frac{(3 \times 13)}{(5 \times 6)} = \frac{(4 \times 12)}{(3 \times 6)} = \frac{39 \times 48}{30 \times 18}$$

The change gears are thus:

A = 39 teeth	*C* = 48 teeth
B = 30 teeth	*D* = 18 teeth

These are gears available in the set listed in Table V (Page 57).

The change gears for the lead of the large helical gear are obtained by following a similar procedure.

Gears in the dividing head driving mechanism must be set to provide the proper combination between the directions of blank rotation and table feed, in relation to the "hand" of helix of the teeth to be milled, as in the case of milling cutters.

Helical gears for parallel shafts have helices of opposite "hands." Hence, after milling one gear, the set-up must be altered for milling the teeth of the second gear. This includes transposing the above gears and reversing the direction of table swivel.

Angle of Table Swivel. The angle of table swivel is the same as the helix angle of the gear teeth. This is calculated at the pitch diameter of the gear, and in the present example is 25° 50′.

When the gear blank is centered on the gear cutter, the table of the machine is swiveled *clockwise* to mill a left hand helix, and *counter-clockwise* when milling a right hand helix.

If the teeth of the small gear are milled on a left hand helix, those in the large gear will be milled on a right hand helix, and the set-up must be changed accordingly.

Indexing. The teeth are indexed into position by plain indexing. From Formula 2 (Page 13), the number of turns of the index crank to index each tooth into position for milling the *small gear* is:

$$t = \frac{40}{24} = 1\,\frac{16}{24}$$

and the number of turns to mill the *large gear* is:

$$t = \frac{40}{48} = \frac{20}{24}$$

The 24-hole circle is used in both cases.

If *block indexing* is used, the gear can be divided into eight blocks. After milling the first tooth, the blank is indexed by turning the index crank 40/8 = 5 turns, to mill a tooth spaced 45° from the first tooth. This indexing is continued until the first tooth is again in the starting position.

The blank is now indexed to mill the tooth following the first tooth by indexing one full turn and 16 spaces on the 24-hole circle. This is followed by indexing 5 turns to mill each successive tooth in the blocks. This indexing is continued until all the teeth in the gear have been milled.

The same procedure is used for milling the 48 teeth in the large gear. Each tooth is positioned by indexing 20 spaces on the 24-hole circle.

Cutting Speed and Feed. The cutting speed and feed rates are selected in relation to the work and cutter materials and the type of cutter used. Gear cutters are usually made of high speed steel, and are of the form relieved type.

Milling Helical Gears for Shafts at Right Angles

EXAMPLE 5: The milling of helical gears for *shafts at right angles* does not present any different problem than the milling of helical gears for *parallel shafts*. The two gears have different helix angles, but of the same "hand" of helix. The procedure used is shown in the following example:

Milling the teeth of helical gears for operation on shafts at right angles (Figure 18). Material: S.A.E. 3115 steel.

Gear Specifications

	Large Gear	*Small Gear*
Number of teeth	25	5
Helix angle	26° 46′	63° 14′
"Hand" of helix	Left hand	Left hand
Normal diametral pitch	8	8
Normal pressure angle	20°	20°
Full depth of tooth	0.2696 in.	0.2696 in.
Pitch diameter	3.496—3.4955 in.	1.3888 in.
Lead at pitch diameter	21.7987 in.	2.200 in.
Center distance between gears	2.4424—2.4422 in.	

Selection of Set-Up. The work is performed on a plain horizontal knee-and-column type milling machine, equipped with a universal spiral milling attachment which permits swiveling or tilting the cutter to the required helix angle, and a universal dividing head driven by the standard driving mechanism (Figure 19).

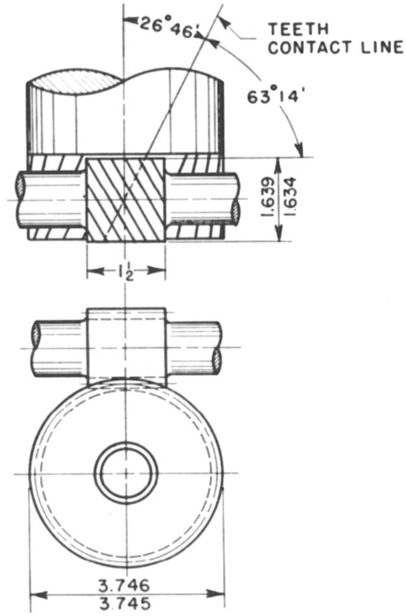


Figure 18 — Helical gears with shafts at right angles.

Selection of Gear Cutter. The number of teeth for which the gear cutter should be selected is obtained from Formula 20 (Page 28):

$$N_c = \frac{25\,(1 + \cos 26^\circ\, 46')}{2 \cos^2 26^\circ\, 46'}$$

$$= \frac{25\,(1 + 0.893)}{2 \times 0.893}$$

$$= 30$$

This indicates that the job requires use of a No. 4, 8 diametral pitch gear cutter (Table **II**, Page 56) made to cut a range of gear teeth from 26 to 34.

If Formula 22 (Page 29) is used, the number of teeth for which the cutter should be selected is then:

$$N_c = \frac{25}{\cos^3 26^\circ\, 46'}$$

$$= \frac{25}{0.893^3}$$

$$= 35$$

and the cutter would be a No. 3 gear cutter made to cut from 35 to 54 teeth (Table **II**, Page 56).

Angle of Swivel for the Milling Cutter. The angle of swivel is the same as the helix angle. Hence, the spindle head of the universal spiral milling attachment is swiveled to the angle of 26° 46′, which can be read from zero mark on the graduated swivel dial of the spindle head.

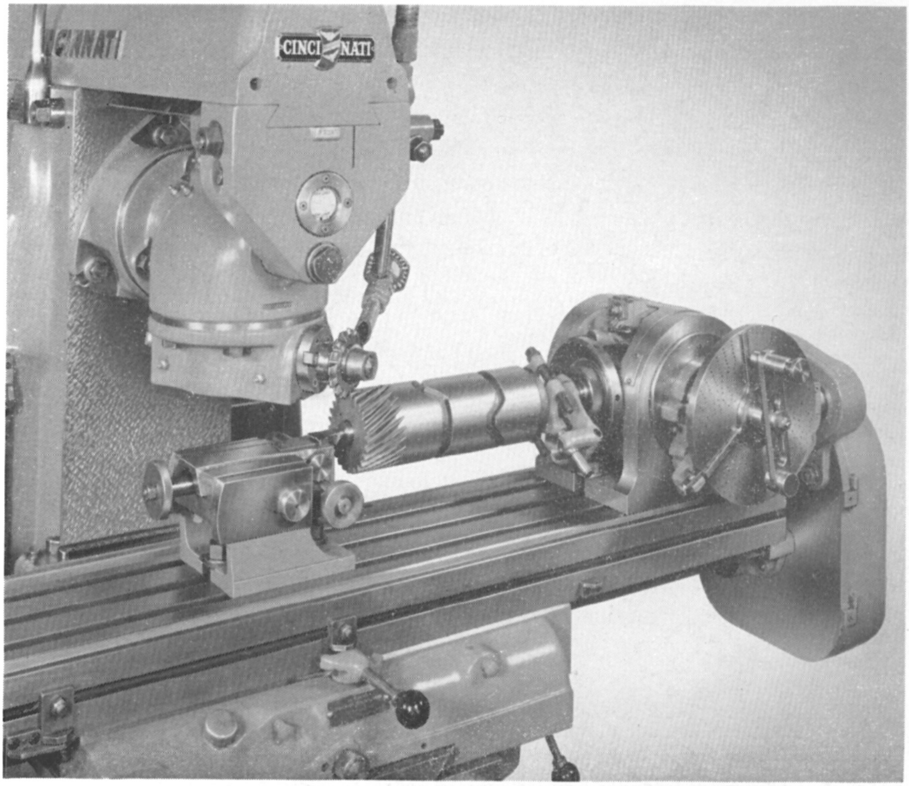


Figure 19—Milling a helical gear on a plain knee-and-column type milling machine equipped with a universal spiral milling attachment and a universal dividing head.

The blank is centered on the cutter by means of radial and axial lines scribed on the blank, and a straight edge held against the side of the cutter.

A centering gage can also be used to increase the accuracy of the alignment (Figure 20). The centering gage consists of a plunger with a centerpoint on one end and a V-notch on the opposite end. It is held on the spindle head by means of a bracket. The plunger is adjusted vertically so that the sides of the V-notch are in contact with the periphery of the cutter. The bracket is then locked in position by means of thumb screw *A*. The centerpoint is lowered and locked by means of screw *B*, and the blank is then adjusted until the scribed mark is aligned with the centerpoint.

Change Gears for Lead. The lead of the large gear is 21.7987 in. The change gears for the standard driving mechanism are calculated as follows:

$$\frac{A \times C}{B \times D} = \frac{21.7987}{10}$$

$$= \frac{21.7987}{100000}$$

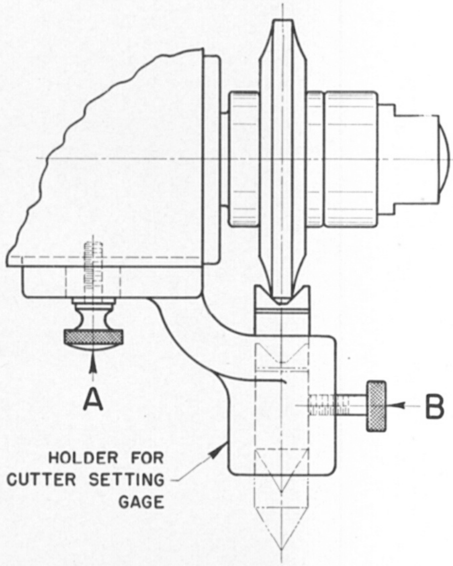


Figure 20—Centering gage.

By means of continuous fractions, it is found that the common fraction 85/39 has a ratio of 2.17948, which corresponds to a lead of 21.7948 in., or 0.0039 in. smaller than the given lead. The change gears for the driving mechanism are now determined from the new fraction:

$$\frac{A \times C}{B \times D} = \frac{95}{39}$$

$$= \frac{5 \times 17}{3 \times 13} = \frac{(5 \times 6) \times (17 \times 3)}{(3 \times 6) \times (13 \times 3)}$$

$$= \frac{30 \times 51}{18 \times 39}$$

hence, the change gears (Table V, Page 57) are:

A = 30 teeth C = 51 teeth

B = 18 teeth D = 39 teeth

Selection of Circle of Holes for Indexing. The number of turns t of the index crank (Formula 2, Page 13) to index the 25 teeth are:

$$t = \frac{40}{25} = 1\ \frac{15}{25}$$

One turn and 15 spaces on the 25-hole circle in the standard index plate (Table III, Page 56) will be required to place each tooth in position for milling.

Depth of Cut. The depth of cut is 0.2696 in., corresponding to the full depth of the teeth. This operation should be carried out by first taking a *roughing cut,* this to be followed by a ***finishing cut.***

Cutting Speed and Feed. The cutting speed and feed are selected in relation to the work and cutter materials and the type of cutter used. The cutting speed for the finishing cut is approximately 50 per cent higher than that used in the roughing cut.

The small gear teeth are cut by following the same procedure as used in milling the teeth for the large gear.

WORMS AND WORM WHEELS

Worms and worm wheels are used in drives to obtain a great reduction in speed ratio between the worm and worm wheel. The worm is the *driver*; the worm wheel is the ***driven member.*** This type of gearing arrangement is known also as an "endless screw." The ratio of the drive is independent of the relative pitch diameters of the worm and worm wheel.

The *worm* is a screw with a single thread or multiple threads, of such a form that its axial cross-section is the same as that of a rack. The teeth of the *worm wheel* are of a special form required to provide proper meshing conditions with the worm.

If the worm, has a single thread, the ratio of the drive is equal to the number of teeth in the worm wheel. With a constant number of teeth in the worm wheel, the drive ratio decreases as the number of threads in the worm is increased. With double and quadruple thread worms, for example, the drive ratio becomes one-half and one-fourth, respectively, of the number of teeth in the worm wheel.

In the worm, the distance between the centers of two adjacent threads is termed the ***pitch.*** The *lead* is the distance which any one thread advances in one revolution of the worm. Therefore, the lead and pitch in a single thread worm are equal; in double and quadruple thread worms, the lead is, respectively, twice and four times the pitch.

Cutting Worm Wheel Teeth on a Milling Machine. Worms and worm wheels can be cut on a milling machine by means of *thread milling cutters* and ***hobs.*** The procedure used is illustrated in the following examples:

Gashing and Hobbing a Worm Wheel

EXAMPLE 6: Milling a 100-tooth, right hand, cast iron worm wheel. Pitch diameter: 7.9576 in. Circular pitch: 0.250 in. Full depth: 0.1716 in. Center distance: 4.7743 in. Gashing angle: 2° 52′. Pitch diameter of worm: 1.5916 in. (Figure 21).

In cutting the teeth of a worm wheel on a milling machine, two operations are required: (1) *gashing* the teeth, and (2) *hobbing* the teeth to the correct size and shape.

Selection of Set-Up. A universal type milling machine equipped with a universal dividing head is used for this operation. The blank is placed between centers of the dividing head and tailstock.

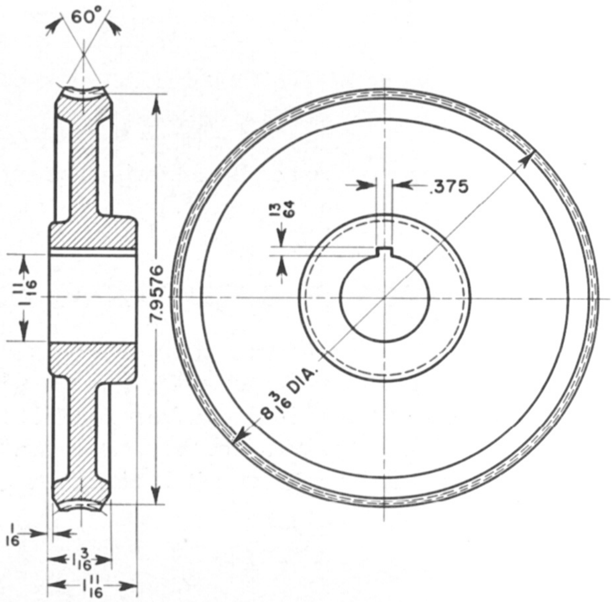


Figure 21—Dimensions of a worm wheel.

Gashing the Teeth—Selection of Cutter. The gashing operation consists of *roughing* the gear teeth with an involute gear cutter having the same pitch and diameter as the worm.

The table of the machine is swiveled to the gashing angle of 2° 52′, in a counter-clockwise direction for a right hand worm, after aligning the blank on the center of the gashing cutter both crosswise and longitudinally. The gashing operation is then performed by feeding the work vertically to the depth of the teeth. It is necessary, however, to leave a sufficient amount of stock for the *finishing* operation.

If not given, the gashing angle can be calculated from Formula 9 (Page 25), using the known values of the lead and pitch diameter of the worm. The gashing angles for worm wheels for a variety of worms from 5/8″ to 6″ diameter and from 1/10″ to 1½″ lead may be taken directly from the Table **VI**, (Pages 58 and 59).

In the present worm wheel, the pitch diameter of the worm is 1.5916 in. and the lead is 0.250 in. Hence:

$$\tan E = \frac{0.250}{3.14 \times 1.59} \ .05$$

and:

$$E = 2° \ 52'$$

Indexing. Plain indexing is used for positioning each tooth of the worm wheel. The correct circle of holes on the index plate and number of turns of the index crank are found by means of Formula 2 (Page 13):

$$t = \frac{40}{100} = \frac{2}{5}$$

$$= \frac{12}{30}$$

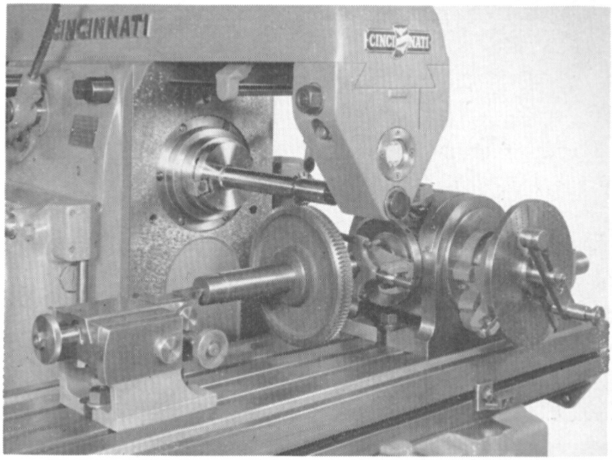


Figure 22—Set-up for gashing the teeth in a worm wheel.

Each tooth is positioned by indexing 12 spaces on the 30-hole circle of the standard plate (Table **III**, Page 56). The set-up for gashing the teeth on the worm wheel is illustrated in Figure 22.

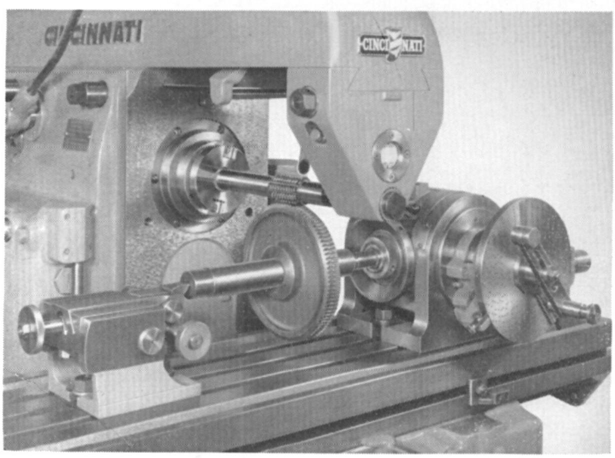


Figure 23—Set-up for hobbing the teeth of a worm wheel on a milling machine.

Hobbing the Worm Wheel Teeth. For the hobbing operation, the worm wheel is held between centers but free from the dividing head driving dog, *thus allowing the hob to drive the wheel while the teeth are cut.*

The worm wheel axis is at right angles to

that of the worm. It is therefore necessary to set the table of the universal machine in the usual straight position, so that the axis of the worm wheel is at right angles to the arbor on which the hob is mounted (Figure 23).

The *hob* is made for a ¼ in. pitch, ¼ in. lead, right hand single thread. The diameter of the hob is 1¾ in., the same as the outside diameter of the worm.

The workpiece is adjusted so that the hob centers over the rim of the worm wheel. The table of the machine is locked in position to prevent its moving while the teeth are being hobbed. The work is then raised gradually until the proper depth is obtained.

If a large amount of stock is to be removed or an exceptionally good finish is required, the worm wheel is passed under the hob a number of times, bringing it into the *final depth* for the last revolution.

Milling Worms

EXAMPLE 7: Milling a right hand, single thread worm for the worm wheel shown in Figure 21. The worm dimensions are shown in Figure 24. The specifications are as follows:

Outside diameter = 1.750 in.	Lead = 0.250 in.
Pitch diameter = 1.591 in.	Pitch = 0.250 in.

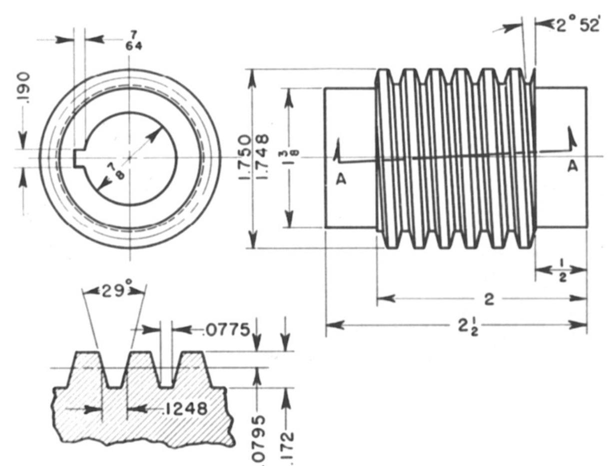


Figure 24—Dimensions of the worm for the worm wheel shown in Figure 21.

Selection of Set-Up. The worm can be milled on a milling machine with *thread milling cutters.* The cutter is mounted on the spindle of a universal spiral milling attachment, and is swiveled to the helix angle of the worm threads. The worm is placed between the centers of a dividing head and tailstock.

The set-up for this operation is made on a universal knee-and-column type milling machine. The universal dividing head has a wide range divider, and is driven by means of a ***short lead attachment*** to obtain the $\frac{1}{4}$ in. lead of the worm thread (**Figure 25**).

Selection of Cutter. The ***thread milling cutter*** is selected for the given $\frac{1}{4}$ in. pitch and 29° included angle of the worm threads. Since the worm has a single thread, a thread milling cutter of stock size can be used. This cutter will have a 1 in. hole diameter and an outside diameter of $2\frac{5}{8}$ in. The cutter is swiveled to the angle of 2° 52′, and is then centered on the worm blank.

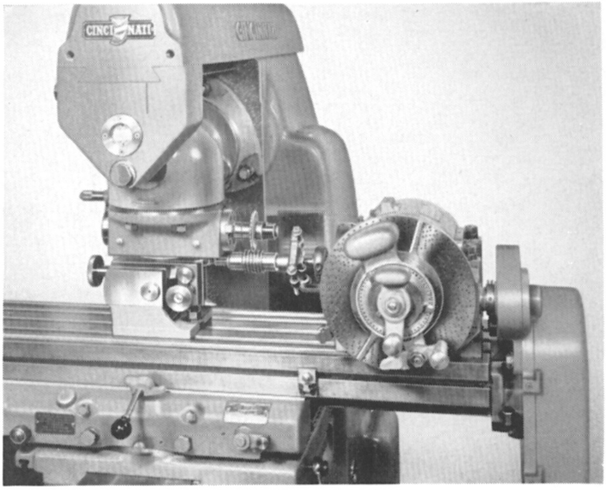


Figure 25—Milling a worm with a universal spiral milling attachment with wide range divider, and a short lead attachment.

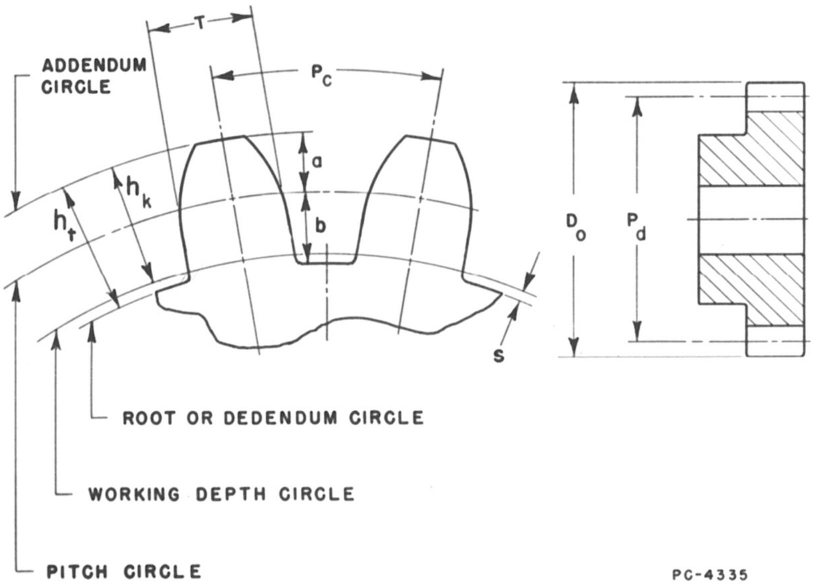
NOTES

GEAR FORMULAE AND REFERENCE DATA

INDEX

RULES AND FORMULAE FOR SPUR GEAR CALCULATIONS

(Circular Pitch)



The following symbols are used in conjunction with the formulae for determining the proportions of spur gear teeth.

P = Diametral pitch.
P_c = Circular pitch.
P_d = Pitch diameter.
D_o = Outside diameter.
N = Number of teeth in the gear.
T = Tooth thickness.
a = Addendum.
b = Dedendum.
h_k = Working depth.
h_t = Whole depth.
S = Clearance.
C = Center distance.
L = Length of rack.

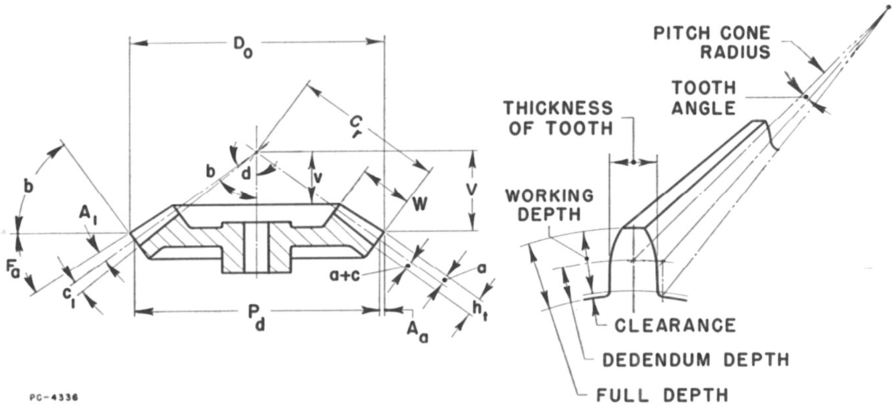
RULES AND FORMULAE FOR SPUR GEAR CALCULATIONS—Continued

(Circular Pitch)

TO FIND	RULE	FORMULA
Diametral pitch P	Divide 3.1416 by the circular pitch.	$P = \frac{3.1416}{P_c}$
Circular pitch P_c	Divide 3.1416 by the diametral pitch.	$P_c = \frac{3.1416}{P}$
Pitch diameter P_d	Divide the number of teeth by the diametral pitch.	$P_d = \frac{N}{P}$
Outside diameter D_o	Add 2 to the number of teeth and divide the sum by the diametral pitch.	$D_o = \frac{N + 2}{P}$
Number of teeth N	Multiply the pitch diameter by the diametral pitch.	$N = P_d\, P$
Tooth thickness T	Divide 1.5708 by the diametral pitch.	$T = \frac{1.5708}{P}$
Addendum a	Divide 1.0 by the diametral pitch.	$a = \frac{1.0}{P}$
Dedendum b	Divide 1.157 by the diametral pitch.	$b = \frac{1.157}{P}$
Working depth hk	Divide 2 by the diametral pitch.	$hk = \frac{2}{P}$
Whole depth ht	Divide 2.157 by the diametral pitch.	$ht = \frac{2.157}{P}$
Clearance S	Divide 0.157 by the diametral pitch.	$S = \frac{0.157}{P}$
Center distance C	Add the number of teeth in both gears and divide the sum by two times the diametral pitch.	$C = \frac{N_1 + N_2}{2P}$
Length of rack L	Multiply the number of teeth in the rack by the circular pitch.	$L = N\, P_c$

RULES AND FORMULAE FOR BEVEL GEAR CALCULATIONS

(Shafts at Right Angles)



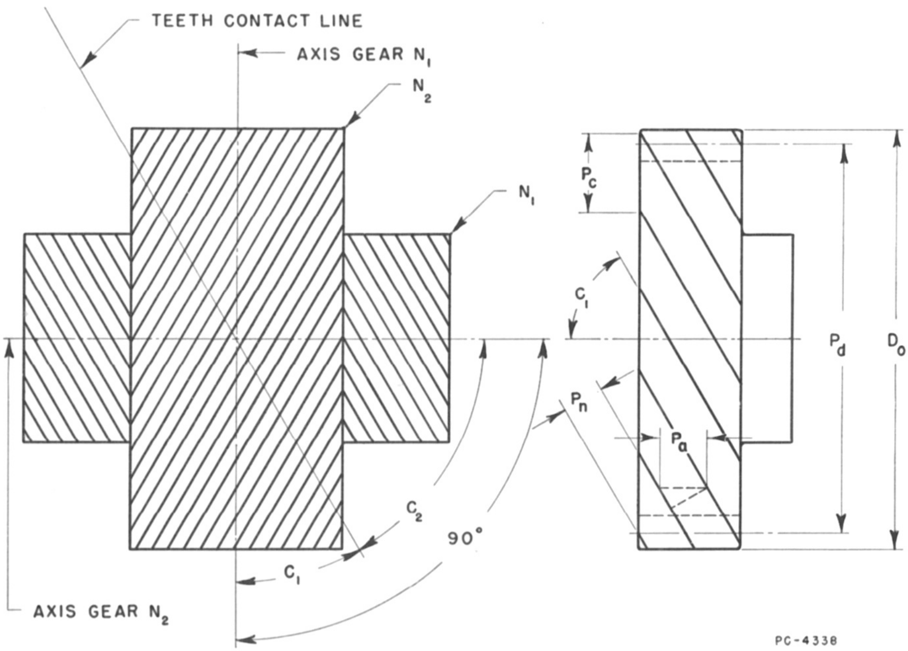
The following symbols are used in conjunction with the formulae for determining the proportions of bevel gear teeth.

P = Diametral pitch.
P_c = Circular pitch.
P_d = Pitch diameter.
b = Pitch cone angle.
C_r = Pitch cone radius.
a = Addendum.
A_1 = Addendum angle.
A_a = Angular addendum.
D_o = Outside diameter.
c_1 = (dedendum plus clearance) angle.
$a+c$ = Dedendum plus clearance.
a_s = Addendum of small end of tooth.
T_L = Thickness of tooth at pitch line.
T_s = Thickness of tooth at pitch line at small end of gear.
F_a = Face angle.
h_t = Whole depth of tooth space.
V = Apex distance at large end of tooth.
v = Apex distance at small end of tooth.
m_g = Gear ratio.
N = Number of teeth.
N_g = Number of teeth in gear.
N_p = Number of teeth in pinion.
d = Cutting angle.
W = Width of gear tooth face.
N_c = Number of teeth of imaginary spur gear for which cutter is selected.

RULES AND FORMULAE FOR BEVEL GEAR CALCULATIONS—Continued
(Shafts at Right Angles)

TO FIND	RULE	FORMULA
Diametral pitch P	Divide the number of teeth by the pitch diameter.	$P = \frac{N}{Pd}$
Circular pitch Pc	Divide 3.1416 by the diametral pitch.	$Pc = \frac{3.1416}{P}$
Pitch diameter Pd	Divide the number of teeth by the diametral pitch.	$Pd = \frac{N}{P}$
Pitch cone angle of pinion $\tan bp$	Divide the number of teeth in the pinion by the number of teeth in the gear to obtain the tangent.	$\tan bp = \frac{Np}{Ng}$
Pitch cone angle of gear $\tan bg$	Divide the number of teeth in the gear by the number of teeth in the pinion to obtain the tangent.	$\tan bg = \frac{Ng}{Np}$
Pitch cone radius Cr	Divide the pitch diameter by twice the sine of the pitch cone angle.	$Cr = \frac{Pd}{2\ (\sin b)}$
Addendum a	Divide 1.0 by the diametral pitch.	$a = \frac{1.0}{P}$
Addendum angle $\tan A_1$	Divide the addendum by the pitch cone radius to obtain the tangent.	$\tan A_1 = \frac{a}{Cr}$
Angular addendum Aa	Multiply the addendum by the cosine of the pitch cone angle.	$Aa = a \cos b$
Outside diameter Do	Add twice the angular addendum to the pitch diameter.	$Do = Pd + 2Aa$
(Dedendum plus clearance) angle $\tan c_1$	Divide the dedendum plus clearance by the pitch cone radius to obtain the tangent.	$\tan c_1 = \frac{a + c}{Cr}$
Addendum of small end of tooth as	Subtract the width of face from the pitch cone radius, divide the remainder by the pitch cone radius and multiply by the addendum.	$as = a\left(\frac{Cr - W}{Cr}\right)$
Thickness of tooth at pitch line T_L	Divide the circular pitch by 2.	$T_L = \frac{Pc}{2}$
Thickness of tooth at pitch line at small end of gear Ts	Subtract the width of face from the pitch cone radius, divide the remainder by the pitch cone radius and multiply by the thickness of the tooth at the pitch line.	$Ts = T_L\left(\frac{Cr - W}{Cr}\right)$
Face angle Fa	Subtract the sum of the pitch cone and addendum angles from 90°.	$Fa = 90° - (b + A_1)$
Whole depth of tooth space ht	Divide 2.157 by the diametral pitch.	$ht = \frac{2.157}{P}$
Apex distance at large end of tooth V	Multiply one-half the outside diameter by the tangent of the face angle.	$V = \left(\frac{Do}{2}\right) \tan Fa$
Apex distance at small end of tooth v	Subtract the width of face from the pitch cone radius, divide the remainder by the pitch cone radius and multiply by the apex distance.	$v = V\left(\frac{Cr - W}{Cr}\right)$
Gear ratio mg	Divide the number of teeth in the gear by the number of teeth in the pinion.	$mg = \frac{Ng}{Np}$
Number of teeth in gear and/or pinion Ng, Np	Multiply the pitch diameter by the diametral pitch.	$Ng = Pd\,P$ $Np = Pd\,P$
Cutting angle d	Subtract the dedendum plus clearance angle from the pitch cone angle.	$d = b - c_1$
Number of teeth of imaginary spur gear for which cutter is selected Nc	Divide the number of teeth in actual gear by the cosine of the pitch cone angle.	$Nc = \frac{N}{\cos b}$

RULES AND FORMULAE FOR HELICAL GEAR CALCULATIONS



The following symbols are used in conjunction with the formulae for determining the proportions of helical gear teeth.

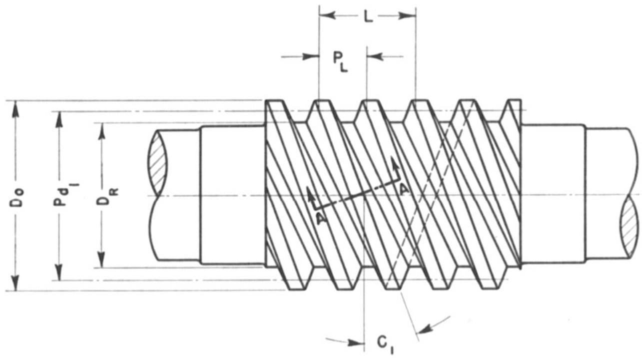
P_{nd} = Normal diametral pitch (pitch of cutter).
P_c = Circular pitch.
P_a = Axial pitch.
P_n = Normal pitch.
P_d = Pitch diameter.
S = Center distance.
C, C_1, C_2 = Helix angle of the gears.
L = Lead of tooth helix.
T_n = Normal tooth thickness at pitch line.
a = Addendum.
h_t = Whole depth of tooth.
N, N_1, N_2 = Number of teeth in the gears.
D_o = Outside diameter.
N_c = Hypothetical number of teeth for which the gear cutter should be selected.

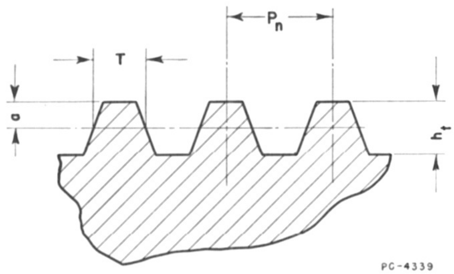
RULES AND FORMULAE FOR HELICAL GEAR CALCULATIONS—Continued

TO FIND	RULE	FORMULA
Normal diametral pitch P_{nd}	Divide the number of teeth by the product of the pitch diameter and the cosine of the helix angle.	$P_{nd} = \dfrac{N}{P_d \cos C_1}$
Circular pitch P_c	Multiply the pitch diameter of the gear by 3.1416, and divide the product by the number of teeth in the gear.	$P_c = \dfrac{3.1416\ P_d}{N}$
Axial pitch P_a	Multiply the circular pitch by the cotangent of the helix angle.	$P_a = P_c \cot C_1$
Normal pitch P_n	Divide 3.1416 by the normal diametral pitch.	$P_n = \dfrac{3.1416}{P_{nd}}$
Pitch diameter P_d	Divide the number of teeth by the product of the normal pitch and the cosine of the helix angle.	$P_d = \dfrac{N}{P_{nd} \cos C_1}$
Center distance S	Divide the sum of the pitch diameters of the mating gears by 2.	$S = \dfrac{P_{d_1} + P_{d_2}}{2}$
Checking Formulae (shafts at right angles)	Multiply the number of teeth in the first gear by the tangent of the tooth angle of that gear, and add the number of teeth in the second gear to the product. The sum should equal twice the product of the center distance multiplied by the normal diametral pitch, multiplied by the sine of the helix angle.	$N_1 + (N_2 \tan C_2) = 2\ S\ P_{nd} \sin C_1$
Lead of tooth helix L	Multiply the pitch diameter by 3.1416 times the cotangent of the helix angle.	$L = 3.1416\ P_d \cot C_1$
Normal tooth thickness at pitch line T_n	Divide 1.571 by the normal diametral pitch.	$T_n = \dfrac{1.571}{P_{nd}}$
Addendum a	Divide the normal pitch by 3.1416.	$a = \dfrac{P_n}{3.1416}$
Whole depth of tooth ht	Divide 2.157 by the normal diametral pitch.	$ht = \dfrac{2.157}{P_{nd}}$
Outside diameter D_o	Add twice the addendum to the pitch diameter.	$D_o = P_d + 2\ a$
Hypothetical number of teeth for which gear cutter should be selected N_c	Divide the number of teeth in the gear by the cube of the cosine of the helix angle.	$N_c = \dfrac{N_1}{(\cos C_1)^3}$

RULES AND FORMULAE FOR WORM GEAR CALCULATIONS (Solid Type)

(Single and Double Thread—14½° Pressure Angle)





SECTION A-A
DOUBLE SIZE
NORMAL TO HELIX ANGLE

The following symbols are used in conjunction with the formulae for determining the proportions of worm gear teeth.

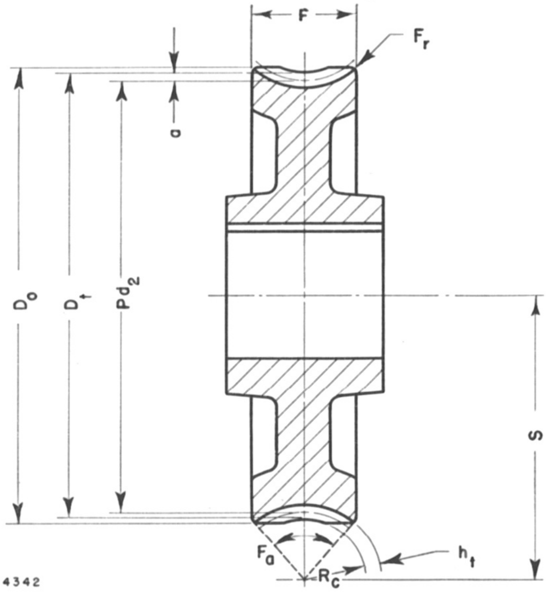
P_L = Linear pitch.
P_{d_1} = Pitch diameter.
D_o = Outside diameter.
N_w = Number of threads.
D_R = Root diameter.
h_t = Whole depth of tooth.
C_1 = Helix angle.
P_n = Normal pitch.
a = Addendum.
L = Lead.
T = Normal tooth thickness.
t = Width of thread tool at end.

RULES AND FORMULAE FOR WORM GEAR CALCULATIONS (Solid Type)—Continued
(Single and Double Thread—14½° Pressure Angle)

TO FIND	RULE	FORMULA
Linear pitch P_L	Divide the lead by the number of threads in the whole worm; i. e., one if single-threaded or four if quadrupled threaded.	$P_L = \frac{L}{N_w}$
Pitch diameter Pd_1	Subtract twice the addendum from the outside diameter.	$Pd_1 = D_O - 2\,a$
Outside diameter D_O	Add twice the addendum of the worm to the pitch diameter of the worm wheel.	$D_O = Pd_1 + 2\,a$
Root diameter D_R	Subtract twice the whole depth of the tooth from the outside diameter.	$D_R = D_O - 2\,ht$
Whole depth of tooth ht	Multiply the linear pitch by 0.6866.	$ht = 0.6866 P_L$
Helix angle C_1	Multiply the pitch diameter of the worm by 3.1416, and divide the product by the lead. The quotient is the cotangent of the helix angle.	$\cot C_1 = \frac{3.1406 Pd_2}{L}$
Normal pitch P_n	Multiply the linear pitch by the cosine of the helix angle of the worm.	$P_n = P_L \cos C_1$
Addendum a	Multiply the linear pitch by 0.3183.	$a = 0.3183\,P_L$
Lead L	Multiply the linear pitch by the number of threads.	$L = P_L N_w$
Normal tooth thickness T	Multiply one-half the linear pitch by the cosine of the helix angle.	$T = \frac{P_L}{2} \cos C_1$
Width of thread tool at end t	Multiply the linear pitch by 0.31.	$t = 0.31\,P_L$

RULES AND FORMULAE FOR WORM WHEEL CALCULATIONS

(Single and Double Thread—14½° Pressure Angle)



PC-4342

The following symbols are used in conjunction with the formulae for determining the proportions of worm wheel teeth.

P_c = Circular pitch.
P_{d_2} = Pitch diameter.
N = Number of teeth.
D_o = Outside diameter.
D_t = Throat diameter.
R_c = Radius of curvature of worm wheel throat.
D = Diameter to sharp corners.
F_a = Face angle.
F = Face width of rim.
F_r = Radius at edge of face.
a = Addendum.
h_t = Whole depth of tooth.
S = Center distance between worm and worm wheel.
G = Gashing angle.

RULES AND FORMULAE FOR WORM WHEEL CALCULATIONS—Continued

(Single and Double Thread—14½° Pressure Angle)

TO FIND	RULE	FORMULA
Circular pitch P_c	Divide the pitch diameter by the product of 0.3183 and the number of teeth.	$P_c = \frac{P_{d_2}}{0.3183\ N}$
Pitch diameter P_{d_2}	Multiply the number of teeth in the worm wheel by the linear pitch of the worm, and divide the product by 3.1416.	$P_{d_2} = \frac{NP_L}{3.1416}$
Outside diameter D_o	Multiply the circular pitch by 0.4775 and add the product to the throat diameter.	$D_o = D_t + 0.4775\ P_c$
Throat diameter D_t	Add twice the addendum of the worm tooth to the pitch diameter of the worm wheel.	$D_t = P_{d_2} + 2\ a$
Radius of curvature of worm wheel throat R_c	Subtract twice the addendum of the worm tooth from half the outside diameter of the worm.	$R_c = \frac{D_o}{2} - 2\ a$
Diameter to sharp corners D	Multiply the radius of curvature of the worm-wheel throat by the cosine of half the face angle, subtract this quantity from the radius of curvature. Multiply the remainder by 2, and add the product to the throat diameter of the worm wheel.	$D = 2\ (R_c - R_c \times \cos \frac{F_a}{2}) + D_t$
Face width of rim F	Multiply the circular pitch by 2.38 and add 0.25 to the product.	$F = 2.38\ P_c + 0.25$
Radius at edge of face F_r	Divide the circular pitch by 4.	$F_r = \frac{P_c}{4}$
Addendum a	Multiply the circular pitch by 0.3183.	$a = 0.3183\ P_c$
Whole depth of tooth h_t	Multiply the circular pitch by 0.6866.	$h_t = 0.6866\ P_c$
Center distance between worm and worm wheel S	Add the pitch diameter of the worm to the pitch diameter of the worm wheel and divide the sum by 2.	$S = \frac{P_{d_1} + P_{d_2}}{2}$
Gashing angle G	Divide the lead of the worm by the circumference of the pitch circle. The result will be the cotangent of the gashing angle.	$\cot G = \frac{L}{3.1416\ d}$

TABULAR DATA

Table I

Change Gears for Rack Indexing Attachment

Number of Teeth						
28	35	42	44	49	56	63
70	77	84	88	91	98	105

Table **II** lists the numbers of standard involute gear cutters and the corresponding ranges of gear teeth which they are made to cut, as given by cutter manufacturers:

Table II

Standard Involute Gear Cutters

Cutter Number	1	2	3	4
Range of Gear Teeth Cut	135 to a Rack	55 to 134	35 to 54	26 to 34
	21 to 25	17 to 20	14 to 16	12 or 13
Cutter Number	5	6	7	8

The standard side index plate of the universal dividing head is provided with a number of circles of holes on both sides, in the combinations shown in Table **III**:

Table III

Circles of Holes in Standard Side Index Plate

One Side	24–25–28–30–34–37–38–39–41–42–43
Other Side	46–47–49–51–53–54–57–58–59–62–66

TABULAR DATA—Continued

The high number index plates (Table IV) have a greater number of circles of holes and a greater range of holes in the circles than the standard side index plate (Table III). Thus, they provide a substantially greater selection for indexing jobs than the standard index plate.

Table IV

Circles of Holes in High Number Side Index Plates

Plate No. 1	Side A	30–48–69–91–99–117–129–147–171–177–189
	Side B	36–67–81–97–111–127–141–157–169–183–199
Plate No. 2	Side C	34–46–79–93–109–123–139–153–167–181–197
	Side D	32–44–77–89–107–121–137–151–163–179–193
Plate No. 3	Side E	26–42–73–87–103–119–133–149–161–175–191
	Side F	28–38–71–83–101–113–131–143–159–173–187

Table V

Change Gears for Standard Enclosed Driving Mechanism

Quantity	No. of Teeth	Quantity	No. of Teeth
1	17	1	33
1	18	1	36
1	19	1	39
1	20	1	42
1	21	1	45
1	22	1	48
2	24	1	51
1	27	1	55
1	30	1	60

TABULAR DATA—Continued

Table VI

Gashing Angles for Worm Wheels

Lead of Worm in Inches	No. of Threads per Inch in Worm	PITCH DIAMETER OF WORM															
		5/8	3/4	7/8	1	1 1/8	1 1/4	1 3/8	1 1/2	1 5/8	1 3/4	1 7/8	2	2 1/8	2 1/4	2 3/8	2 1/2
1/10	10	2°55′	2°26′	2° 5′	1°49′	1°37′	1°28′	1°20′	1°13′	1° 7′	1° 2′	58′	55′	52′	49′	46′	44′
1/9	9	3°14′	2°42′	2°19′	2° 1′	1°48′	1°37′	1°28′	1°21′	1°15′	1° 9′	1° 5′	1° 1′	57′	54′	51′	49′
1/8	8	3°38′	3° 2′	2°36′	2°17′	2° 2′	1°49′	1°39′	1°31′	1°24′	1°18′	1°13′	1° 8′	1° 4′	1° 1′	58′	54′
1/7	7	4°10′	3°28′	2°58′	2°36′	2°19′	2° 5′	1°54′	1°44′	1°36′	1°29′	1°23′	1°18′	1°14′	1° 9′	1° 6′	1° 3′
1/6	6	4°51′	4° 3′	3°28′	3° 2′	2°42′	2°26′	2°13′	2° 1′	1°52′	1°44′	1°37′	1°31′	1°26′	1°21′	1°17′	1°13′
1/5	5	5°49′	4°51′	4°10′	3°39′	3°14′	2°55′	2°39′	2°26′	2°15′	2° 5′	1°57′	1°49′	1°43′	1°37′	1°32′	1°27′
1/4	4	7°16′	6° 4′	5°12′	4°33′	4° 3′	3°39′	3°19′	3° 2′	2°48′	2°36′	2°26′	2°17′	2° 9′	2° 2′	1°55′	1°49′
2/7	3 1/2	8°17′	6°55′	5°56′	5°12′	4°37′	4°10′	3°47′	3°28′	3°12′	2°58′	2°47′	2°36′	2°27′	2°19′	2°12′	2° 5′
1/3	3	9°38′	8° 3′	6°55′	6° 3′	5°23′	4°51′	4°25′	4° 3′	3°44′	3°28′	3°14′	3° 2′	2°52′	2°42′	2°33′	2°26′
4/11	2 3/4	10°30′	8°46′	7°32′	6°36′	5°52′	5°17′	4°49′	4°25′	4° 4′	3°47′	3°32′	3°19′	3° 7′	2°57′	2°47′	2°39′
3/8	2 2/3	10°49′	9° 3′	7°46′	6°48′	6° 4′	5°27′	4°58′	4°33′	4°12′	3°54′	3°39′	3°25′	3°13′	3° 2′	2°53′	2°44′
2/5	2 1/2	11°31′	9°38′	8°17′	7°15′	6°27′	5°49′	5°17′	4°51′	4°29′	4°10′	3°53′	3°39′	3°26′	3°14′	3° 4′	2°55′
4/9	2 1/4	......	......	......	8° 3′	7°10′	6°27′	5°52′	5°23′	4°59′	4°37′	4°19′	4° 3′	3°46′	3°36′	3°25′	3°14′
1/2	2	......	......	......	......	......	7°15′	6°36′	6° 3′	5°36′	5°12′	4°51′	4°33′	4°17′	4° 3′	3°50′	3°39′
4/7	1 3/4	......	......	......	......	......	......	......	6°55′	6°23′	5°56′	5°32′	5°12′	4°54′	4°37′	4°23′	4°10′
2/3	1 1/2	......	......	......	......	......	......	......	......	......	......	6°27′	6° 3′	5°42′	5°23′	5° 6′	4°51′
3/4	1 1/3	......	......	......	......	......	......	......	......	......	......	......	......	6°25′	6° 3′	5°44′	5°27′
4/5	1 1/4	......	......	......	......	......	......	......	......	......	......	......	......	......	......	6° 7′	5°49′
1	1	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......
1 1/3	3/4	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......
1 1/2	2/3	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......

TABULAR DATA—Continued

Table VI (Continued)

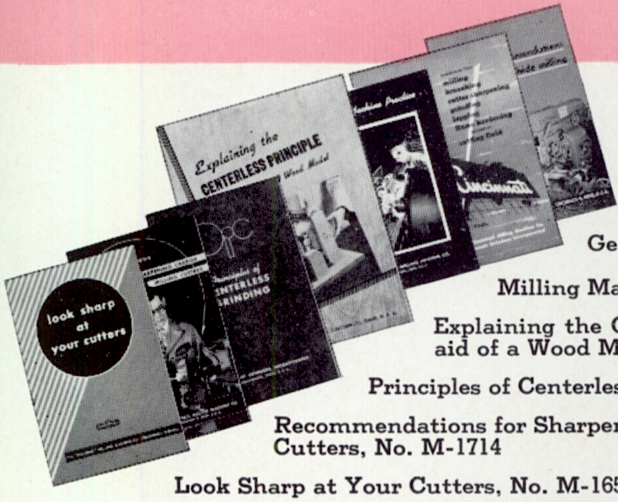
Gashing Angles for Worm Wheels

Lead of Worm in Inches	No. of Threads per Inch in Worm	PITCH DIAMETER OF WORM															
		2⅝	2¾	2⅞	3	3¼	3½	3¾	4	4¼	4½	4¾	5	5¼	5½	5¾	6
1/10	10	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......
1/9	9	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......
1/8	8	52'	50'	48'	46'	42'	39'	36'	34'	32'	......	......	......	......	......	......	......
1/7	7	1°	57'	54'	52'	48'	45'	42'	39'	37'	35'	33'	......	......	......	......	......
1/6	6	1° 9'	1° 6'	1° 3'	1° 1'	56'	52'	48'	46'	43'	40'	38'	36'	35'	......	......	......
1/5	5	1°23'	1°20'	1°16'	1°13'	1° 7'	1° 3'	58'	55'	52'	49'	46'	44'	42'	40'	38'	......
1/4	4	1°44'	1°39'	1°35'	1°31'	1°24'	1°18'	1°13'	1° 8'	1° 4'	1° 1'	58'	55'	52'	50'	48'	46'
2/7	3½	1°59'	1°54'	1°49'	1°44'	1°36'	1°29'	1°23'	1°18'	1°14'	1° 9'	1° 6'	1° 3'	1° 1'	57'	54'	52'
1/3	3	2°19'	2°13'	2° 7'	2° 2'	1°52'	1°44'	1°37'	1°31'	1°26'	1°21'	1°17'	1°13'	1° 9'	1° 6'	1° 3'	1° 1'
4/11	2¾	2°31'	2°25'	2°18'	2°13'	2° 2'	1°54'	1°46'	1°39'	1°34'	1°28'	1°24'	1°20'	1°16'	1°12'	1° 9'	1° 6'
3/8	2⅔	2°36'	2°29'	2°23'	2°17'	2° 6'	1°57'	1°49'	1°43'	1°37'	1°31'	1°26'	1°22'	1°18'	1°15'	1°11'	1° 8'
2/5	2½	2°47'	2°39'	2°32'	2°26'	2°14'	2° 5'	1°57'	1°49'	1°43'	1°37'	1°32'	1°28'	1°23'	1°20'	1°16'	1°13'
4/9	2¼	3° 5'	2°57'	2°49'	2°42'	2°30'	2°19'	2°10'	2° 2'	1°54'	1°48'	1°42'	1°37'	1°33'	1°28'	1°24'	1°21'
1/2	2	3°28'	3°19'	3°10'	3° 2'	2°48'	2°36'	2°26'	2°17'	2° 9'	2° 2'	1°55'	1°49'	1°44'	1°39'	1°35'	1°31'
4/7	1¾	3°58'	3°47'	3°37'	3°28'	3°12'	2°59'	2°47'	2°36'	2°27'	2°19'	2°12'	2° 5'	1°59'	1°54'	1°49'	1°44'
2/3	1½	4°37'	4°25'	4°13'	4° 3'	3°44'	3°28'	3°14'	3° 2'	2°52'	2°42'	2°33'	2°26'	2°19'	2°13'	2° 7'	2° 1'
3/4	1⅓	5°12'	4°58'	4°45'	4°33'	4°12'	3°54'	3°39'	3°25'	3°13'	3° 2'	2°53'	2°44'	2°36'	2°29'	2°23'	2°17'
4/5	1¼	5°32'	5°17'	5° 4'	4°51'	4°20'	4°10'	3°53'	3°39'	3°26'	3°14'	3° 4'	2°55'	2°47'	2°39'	2°32'	2°26'
1	1	6°55'	6°36'	6°19'	6° 3'	5°36'	5°12'	4°51'	4°33'	4°17'	4° 3'	3°50'	3°39'	3°28'	3°19'	3°10'	3° 2'
1⅓	¾	......	......	8°24'	8° 3'	7°26'	6°54'	6°27'	6° 4'	5°42'	5°23'	5° 6'	4°51'	4°37'	4°25'	4°13'	4° 2'
1½	⅔	......	......	......	9° 3'	8°22'	7°46'	7°15'	6°49'	6°26'	6° 4'	6°44'	5°27'	5°12'	4°58'	4°45'	4°33'

NOTES

Additional LITERATURE AVAILABLE

To further assist you in improving your knowledge of milling, grinding and cutter sharpening methods and machines, CINCINNATI offers the additional educational literature shown below. Single copies will be sent to school shop or training instructors free on request. They are available in quantities at a modest charge.



Recommendations for Carbide Milling, No. M-1614

General Catalog, No. M-1712

Milling Machine Practice, No. M-773-1

Explaining the Centerless Principle with the aid of a Wood Model, No. G-509

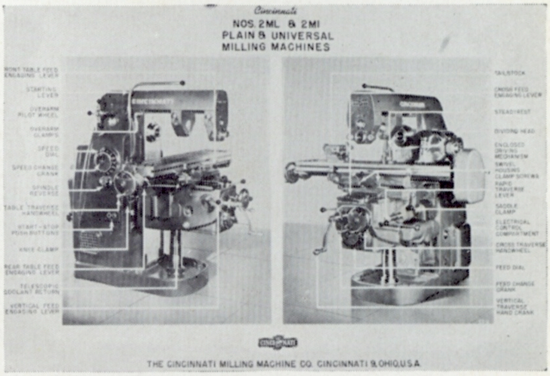
Principles of Centerless Grinding, No. G-503-2

Recommendations for Sharpening Carbide Milling Cutters, No. M-1714

Look Sharp at Your Cutters, No. M-1657-2

Functional diagrams (14″ x 18″) for the following machines will be sent to school shop or training instructors on request.

- No. 2ML and 2MI Milling Machines
- Dial Type Milling Machines
- 0-8 Vertical Milling Machine
- No. 2 Cutter and Tool Grinder
- No. 2 Centerless Grinder
- 12″ Hydraulic Universal Grinder



A casebound text book "A Treatise on Milling and Milling Machines, containing 896 pages of text, over 700 illustrations and charts, is available at $8.00 per copy.

Two 16 mm. color movies "Highway to Production" and "Age of Precision" are available for your showing at no charge.

Please address all requests to the Technical and Service Publications Dept., The Cincinnati Milling Machine Co., 4701 Marburg Ave., Cincinnati 9, Ohio, U. S. A.

THE CINCINNATI MILLING MACHINE CO.
CINCINNATI 9, OHIO, U.S.A.

Publication No. M-1769
Supersedes M-1397

Printed In U.S.A.
5M 452 P